THE ELEMENTS OF

ASTRONOMY FOR SURVEYORS.

mar.

THE ELEMENTS OF ASTRONOMY FOR SURVEYORS.

BY

R. W. CHAPMAN, M.A., B.C.E., F.R.A.S.,

PROFESSOR OF MATHEMATICS AND MECHANICS IN THE UNIVERSITY OF ADELAIDE.

WITH 56 DIAGRAMS.

LONDON:

CHARLES GRIFFIN AND COMPANY, LIMITED.

PHILADELPHIA: J. B. LIPPINCOTT COMPANY.

1919.

PREFACE.

ALTHOUGH there are several excellent books on Surveying that deal more or less thoroughly with astronomical observation, it appeared to the writer, as the result of his experience in teaching the subject, that there is a distinct need of an elementary work suitable for the student and for the surveyor who is taking up astronomical observation for the first time. Most of the purely surveying books are content to quote practical formulæ for the reduction of the observations, with little or no attempt to expound the principles by which the formulæ are derived. On the other hand, the theoretical works on astronomy in which the mathematical theory is developed are generally too recondite for the beginner, and deal to a large extent with matters of no special interest to the surveyor. The present work is an attempt to provide an elementary exposition, not only of the practical methods of observation and computation, but of the main principles that must be thoroughly understood if the surveyor is to be master of his profession. Throughout the work the methods of observation are illustrated with numerous fully worked-out actual observations, and a prominent feature of the book is the attention that is given to the effects of observational and instrumental errors of different kinds. A large proportion of the examples set for working have been taken from the papers set for candidates at the examinations for Licensed Surveyors in Australia.

R. W. C.

ADELAIDE, *September*, 1918.

CONTENTS.

CHAPTER I.

The Solution of Spherical Triangles.

PAGES

A Review of the Principal Formulæ of Spherical Trigonometry, . 1-7

CHAPTER II.

The Celestial Sphere and Astronomical Co-Ordinates.

The Celestial Sphere—The Apparent Motion of the Stars—Definitions of some Fundamental Terms—Astronomical Co-ordinates—Altitude and Azimuth—Right Ascension and Declination — Comparative Advantages of the Two Co-ordinate Systems—The Sidereal Day and Sidereal Time—Hour Angle—Synopsis of Astronomical Terms, . . . 8-20

CHAPTER III.

The Earth.

The Earth as a Globe—Terrestrial Latitude and Longitude—The Zones of the Earth—The Altitude of the Celestial Poles equal to the Latitude of the Place of Observation—To Find the Shortest Distance Between Two Places whose Latitudes and Longitudes are given—The Earth as an Oblate Spheroid—Geographical and Geocentric Latitude—Examples, . . 21-34

CHAPTER IV.

The Sun.

The Sun's Apparent Motion among the Stars—The Earth's Orbit Round the Sun—The Equinoxes—The Sun's Motion in Right Ascension and Declination — The Sun's Semi-Diameter—Plotting the Position of the Sun's Centre on the Celestial Sphere—The Sun's Apparent Annual Path, ... 35-43

CHAPTER V.

TIME.

PAGES

Sidereal Time—Apparent Solar Time—Mean Time—The Three Systems of Time Measurement—Equation of Time—Local Mean Time — Local Sidereal Time — Standard Time—To Change Standard Time to Local Mean Time—To Reduce an Interval of Mean Time to Sidereal Time, and *vice versâ*—To Find Local Sidereal Time at Local Mean Noon, given the Sidereal Time at Mean Noon at Greenwich—Given the Local Mean Time at any instant to Determine the Local Sidereal Time—Given the Sidereal Time to Find the Corresponding Local Mean Time — Alternative Methods for Preceding Problems—Determination of Time of Transit of a known Star across the Meridian—Time of Transit of the First Point of Aries—The Use of the Greenwich Time of Transit of the First Point of Aries in Computations of Local Mean and Sidereal Time—Nautical Almanac Data with Regard to Time Calculations—Examples, ... 44-70

CHAPTER VI.

THE LOCATION OF OBJECTS ON THE CELESTIAL SPHERE.

Given the Right Ascension and Declination of a Star, to Determine its Altitude and Azimuth at any Time — Having Observed the Altitude and Azimuth of a Star, and Noted the Time, to Compute its Right Ascension and Declination—Having Determined the Altitude and Azimuth at a given Time, to Find the Approximate Position of the Star at some Short Interval of Time afterwards—Examples, . . . 71-79

CHAPTER VII.

ASTRONOMICAL AND INSTRUMENTAL CORRECTIONS TO OBSERVATIONS OF ALTITUDE AND AZIMUTH.

Parallax—Horizontal Parallax—Atmospheric Refraction—Corrections to Observations on Account of Residual Instrumental Errors—The Effect of an Error in Collimation—The Elimination of Instrumental Errors by Changing Face—The Error made if the Transverse Axis of the Telescope is not Horizontal—The Use of the Striding Level—Allowance for Error of Alidade Level, ... 80-96

CHAPTER VIII.

THE DETERMINATION OF TRUE MERIDIAN.

PAGES

Referring Marks—First Method: By Equal Altitudes of a Circumpolar Star. Second Method: By a Circumpolar Star at Elongation — Calculation of the Time of Elongation — The Effect of an Error in Latitude—Star Observations in Daylight. Third Method: By Extra-Meridian Observations on Sun or Star—Sun Observations—Computation of Sun's Declination from Nautical Almanac Data—The Effects of Errors in Latitude, Declination and Longitude. Fourth Method: By Time Observations upon a Close Circumpolar Star—Circum-Elongation Observations for Azimuth—Examples, - 97-148

CHAPTER IX.

THE DETERMINATION OF LATITUDE.

First Method: By Meridian Transits—Zenith Pair Observations of Stars — Meridian Altitudes at both Lower and Upper Culminations. Second Method: By Circum-Meridian Observations—Circum-Meridian Observations of the Sun. Third Method: By Prime Vertical Transits—The Effects of Errors in the Time Measurement and in the Setting Out of the Prime Vertical—Striding Level Correction. Fourth Method: By the Altitude of the Pole Star at any Time—A Rough Method by Noting the Rate at which Altitude of Sun or Star changes near the Prime Vertical—A Method by the Measurement of the Horizontal Angle between Two Circumpolar Stars at their Greatest Elongations.—Table for Reduction of Circum-Meridian Observations - Examples, ... 149-181

CHAPTER X.

THE DETERMINATION OF TIME BY OBSERVATION.

First Method: By Meridian Transits—The Effect of an Error in the Direction of the Meridian—The Effect of an Error in the Horizontality of the Transverse Axis—Meridian Transits on both Sides of the Zenith. Second Method: By Extra-Meridian Observations of Sun or Star—Arrangement of the Computation—Averaging several Observations—Observations on both East and West Stars—The Effect of Errors in Latitude, Declination and the Measured Altitude. Third

PAGES

Method: By Equal Altitudes—Error Due to Inequality in the Altitudes—Application of Method to Sun Observations. Fourth Method: Almucantar Method for Time Observations—Sun Dials—The Horizontal Dial—Prime Vertical Dial—Oblique Dials—Time of Rising or Setting of a Celestial Body—Examples, ... 182-218

CHAPTER XI.

THE DETERMINATION OF LONGITUDE.

1. By Portable Chronometers. 2. By Electric Telegraph or Wireless Telegraphy—Recording and Receiving Signals—Comparison of Chronometers—Personal Equation—Programme of Operations. 3. By Flash Lamp Signals—Longitude by Lunar Observations—(*a*) By Lunar Distances—(*b*) By Lunar Culminations—(*c*) By Lunar Occultations, ... 219-235

CHAPTER XII.

THE CONVERGENCE OF MERIDIANS, ... 236-241

INDEX, ... 243-247

ASTRONOMY FOR SURVEYORS.

CHAPTER I.

THE SOLUTION OF SPHERICAL TRIANGLES.

In this chapter the principal formulæ of spherical trigonometry, such as will be afterwards applied to calculations on the celestial sphere, are brought together for convenient reference. No attempt will be made to establish the formulæ, which are demonstrated in any of the ordinary books on spherical trigonometry, but a brief synopsis will be given of the usual methods for the solution of spherical triangles under different conditions.

Great Circles.—The line of intersection made with the surface of a sphere by a plane passing through the centre of the sphere is known as a *great circle*. If this circle passes through two points A and B on the surface of the sphere, then the shortest distance between A and B, measured along the sphere's surface, is that measured along the arc of the great circle joining them. Only one great circle can be drawn to pass through two given points on the surface of a sphere, unless they happen to be at opposite extremities of a diameter, and the length of the shorter arc of this great circle between the two points is the shortest distance between them. Meridians of longitude on the earth's surface are great circles.

In spherical trigonometry it is always assumed that the arcs representing the sides of the triangles considered are arcs of great circles.

Small Circles.—The line of intersection made with the surface of a sphere by a plane that does not pass through the centre is known as a *small circle.* The ordinary formulæ of spherical trigonometry do not apply to triangles having sides that are arcs of small circles. A parallel of latitude on the earth's surface is a small circle. It follows that the shortest distance between two points in the same latitude is not that measured along the parallel of latitude, but is measured along the arc of the great circle joining them.

Spherical Triangles.—Denoting the angles of a spherical triangle by A, B, and C, and the sides opposite to these angles by a, b, and c respectively, the sides being as usual measured by the angles which they subtend at the centre of the sphere, then we have the following fundamental relations :—

(a) The sines of the angles are proportional to the sines of the opposite sides :—

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \quad . \quad . \quad . \quad (1)$$

(b) One side of a triangle is expressed in terms of the two other sides, and the angle included between them by one of the three formulæ :—

$$\left.\begin{aligned} \cos a &= \cos b \cos c + \sin b \sin c \cos A \\ \cos b &= \cos c \cos a + \sin c \sin a \cos B \\ \cos c &= \cos a \cos b + \sin a \sin b \cos C \end{aligned}\right\} \quad . \quad (2)$$

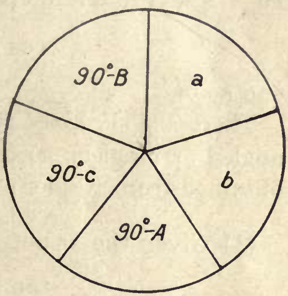
(c) From these may be derived another set of six useful relationships of which the following two are types :—

$$\left.\begin{aligned} \cot a \sin b &= \cot A \sin C + \cos b \cos C \\ \cot b \sin a &= \cot B \sin C + \cos a \cos C \end{aligned}\right\} \quad . \quad (3)$$

Whilst the formulæ (2) and (3) are extremely useful in all sorts of investigations into the properties of spherical triangles, they are not adapted to logarithmic computation, and are consequently not suitable for use in the numerical solution of triangles. For this purpose other formulæ are commonly used, derived from these fundamental relationships but expressed in a form more suitable for use with logarithms.

The Solution of Right-Angled Spherical Triangles.—The relationships between the sides and angles of a right-angled spherical triangle are very conveniently summarised by the mnemonic rules due to Napier, the inventor of logarithms, and known as Napier's Rules of Circular Parts.

Denoting the right angle by C, Napier defines five "circular" parts (*i.e.*, a, b, $90°-A$, $90°-c$, $90°-B$), and these are supposed, as in the figure, to be ranged round a circle in the order in which they stand in the triangle. Then, if any one of these five parts is selected and called the *middle* part, the two parts on each side of it are called the *adjacent* parts, and the remaining two are called the *opposite* parts. For instance, if a is chosen as the middle part, $90°-B$ and b are the adjacent parts, and $90°-c$ and $90°-A$ are the opposite parts. Then Napier's Rules are :—



Sine of *middle* part = product of *tangents* of *adjacent* parts.

Sine of *middle* part = product of *cosines* of *opposite* parts.

Thus

$$\sin a = \cot B \tan b$$

$$\text{and } \sin a = \sin c \sin A.$$

As an aid to memory, it may be noticed that the vowels in the words sine and middle are the same, so with tangent and adjacent, cosine and opposite.

By choosing different parts in turn as the middle parts, we obtain all the possible relationships between the sides and angles, and with a little practice it is easy to choose the particular ones wanted. If we want a relationship between a, b, and c, for example, $90° - c$ must be taken as the middle part, and we have

$$\cos c = \cos a \cos b.$$

If a relationship between a, A, and B is required, take $90° - A$ as the middle part, whence

$$\cos A = \sin B \cos a$$

and so on.

There are six cases to consider in the solution of right-angled triangles, and the formulæ required, readily obtained from Napier's rules, are as follows :—

(1) Given the hypotenuse c and an angle A.

$$\begin{aligned} \tan b &= \tan c \cos A, \\ \cot B &= \cos c \tan A, \\ \sin a &= \sin c \sin A. \end{aligned}$$

(2) Given a side b and the adjacent angle A.

$$\tan c = \frac{\tan b}{\cos A},$$

$$\begin{aligned} \tan a &= \tan A \sin b, \\ \cos B &= \cos b \sin A. \end{aligned}$$

(3) Given the two sides a and b.

$$\begin{aligned} \cos c &= \cos a \cos b, \\ \cot A &= \cot a \sin b, \\ \cot B &= \cot b \sin a. \end{aligned}$$

(4) Given the hypotenuse c and side a,

$$\cos b = \frac{\cos c}{\cos a},$$

$$\cos B = \frac{\tan a}{\tan c},$$

$$\sin A = \frac{\sin a}{\sin c}.$$

(5) Given the two angles A and B,

$$\cos c = \cot A \cot B,$$

$$\cos a = \frac{\cos A}{\sin B},$$

$$\cos b = \frac{\cos B}{\sin A}.$$

(6) Given a side a and opposite angle A,

$$\sin c = \frac{\sin a}{\sin A},$$

$$\sin b = \tan a \cot A,$$

$$\sin B = \frac{\cos A}{\cos a}.$$

The Solution of Oblique Spherical Triangles.—(1) Given the three sides, a, b, and c.

$$\text{Let } s = \tfrac{1}{2}(a + b + c).$$

Then the angle A may be computed from any one of the following three formulæ :—

$$\sin \frac{A}{2} = \sqrt{\frac{\sin (s-b) \,.\, \sin (s-c)}{\sin b \,.\, \sin c}},$$

$$\cos \frac{A}{2} = \sqrt{\frac{\sin s \,.\, \sin (s-a)}{\sin b \,.\, \sin c}},$$

$$\tan A = \sqrt{\frac{\sin (s-b) \,.\, \sin (s-c)}{\sin s \,.\, \sin (s-a)}}.$$

Similar formulæ apply, of course, to the other two angles.

(2) Given two sides a and b, and the included angle C,

$$\tan \tfrac{1}{2} (A+B) = \frac{\cos \frac{1}{2} (a-b)}{\cos \frac{1}{2} (a+b)} \cot \tfrac{1}{2} C,$$

$$\tan \tfrac{1}{2} (A-B) = \frac{\sin \frac{1}{2} (a-b)}{\sin \frac{1}{2} (a+b)} \cot \tfrac{1}{2} C.$$

These determine $\frac{1}{2}(A+B)$ and $\frac{1}{2}(A-B)$, and hence, by addition and subtraction, A and B.

c may either be found from

$$\sin c = \frac{\sin a \sin C}{\sin A},$$

$$\text{or } \tan \tfrac{1}{2} c = \frac{\cos \frac{1}{2} (A+B)}{\cos \frac{1}{2} (A-B)} \tan \tfrac{1}{2} (a+b).$$

The former of the two alternative formulæ for c is the simpler, but as the value of c is here found from its sine, it is sometimes difficult to determine which of two values is to be given to it. This difficulty does not arise with the second formula.

(3) Given one side c and two adjacent angles A and B.

$$\tan \tfrac{1}{2} (a+b) = \frac{\cos \frac{1}{2} (A-B)}{\cos \frac{1}{2} (A+B)} \tan \tfrac{1}{2} c.$$

$$\tan \tfrac{1}{2} (a-b) = \frac{\sin \frac{1}{2} (A-B)}{\sin \frac{1}{2} (A+B)} \tan \tfrac{1}{2} c.$$

These determine $\frac{1}{2}(a+b)$ and $\frac{1}{2}(a-b)$, and hence, by addition and subtraction, a and b.

C may be found either from

$$\sin C = \frac{\sin A \,.\, \sin c}{\sin a}.$$

$$\text{or } \tan \tfrac{1}{2} C = \frac{\sin \frac{1}{2} (a-b)}{\sin \frac{1}{2} (a+b)} \cot \tfrac{1}{2} (A-B).$$

Similar remarks applying to the two formulæ as in case (2).

(4) Given two sides a and b, and the angle opposite one of them A.

This is generally known as the ambiguous case.

B may be found from

$$\sin B = \frac{\sin b}{\sin a} \sin A,$$

which will usually determine two possible values of B. If the value of sin B obtained is greater than unity there will be no solution at all.

Having determined B, C and c may be found from the formulæ :—

$$\tan \tfrac{1}{2} C = \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \tfrac{1}{2}(A+B),$$

$$\tan \tfrac{1}{2} c = \frac{\cos \frac{1}{2}(A+B)}{\cos \frac{1}{2} . (A-B)} \tan \tfrac{1}{2}(a+b).$$

(5) Given two angles A and B, and the side opposite one of them, a.

The solution in this case is similar to case (4), and two solutions are often possible :—

$$\sin b = \frac{\sin B \sin a}{\sin A},$$

after which the same two formulæ as in case (4) determine $\tan \frac{1}{2} C$ and $\tan \frac{1}{2} c$.

Spherical Excess.—The sum of the three angles of a spherical triangle is always greater than 180°, the difference $A + B + C - \pi$ being known as the *spherical excess*.

If this is denoted by E, the area of any spherical triangle $= E r^2$, the spherical excess being in circular measure, and r denoting the radius of the sphere.

CHAPTER II.

THE CELESTIAL SPHERE AND ASTRONOMICAL CO-ORDINATES.

The Celestial Sphere.—We may easily imagine, looking up to the heavens on a cloudless night, that the stars are distributed over the surface of the spherical vault of sky above us. It is not really so, because refined measurements have proved that the distances of the stars differ tremendously, but these distances are so immense that in most cases they cannot be measured even by the most skilful of astronomers with the most delicate of instruments. The consequence is that for practical purposes we are never concerned with the distances of the stars, but only with their directions, and in order to record these it is exceedingly convenient to picture the stars as distributed over the surface of an imaginary spherical sky having its centre at the position of the observer. Thus has arisen the conception of the *Celestial Sphere*, which we may consider as a geometrical device to enable us to record and measure the directions of the stars.

In Fig. 1, suppose that O represents the position of the observer. With O as centre, imagine a spherical surface described with a radius of any length we please; we may make it a few feet or a few thousand miles, it makes no difference. Now, let A, B, and C be three of these immensely distant stars, and let the lines O A, O B, and O C cut our imaginary sphere in *a*, *b*, and *c* respectively. Then, if we are only concerned with the directions of the stars, we may just as well picture them as occupying the positions *a*, *b*, and *c* as their actual places A, B, and C.

In fact, to the observer at O their appearance would be unaltered. So, proceeding in this way, we may picture all the stars in the sky as occupying places on this imaginary surface, which is then known as the *Celestial Sphere*. It may be considered as the spherical surface upon which the stars *appear* to lie, but, of course, in reality they are not all equally distant from us, and they are only represented in this way in order to conveniently measure their directions.

If through the point O a vertical line be drawn to

cent. of earth

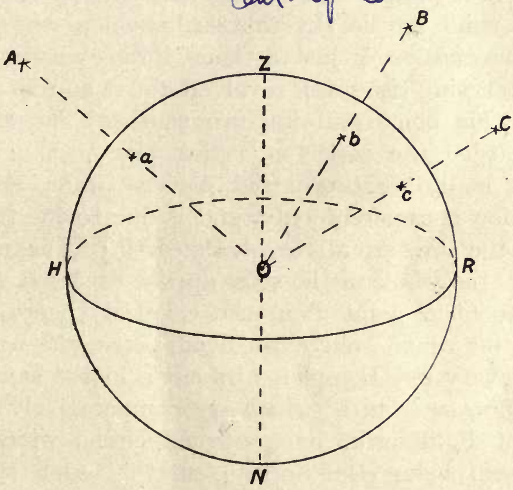


Fig. 1.

intersect the celestial sphere over the observer's head ~~in Z,~~ and to cut it vertically below his feet at N, the point Z is called the *Zenith* and the point N the *Nadir*. The Zenith is thus the point in the celestial sphere directly over the observer.

If a horizontal plane H R be drawn through O, a plane —that is to say, at right angles to the vertical O Z, the direction in which gravity acts—it will cut the celestial sphere in a great circle, which is called the *Celestial Horizon*. To an observer whose eye was close to the surface of a calm ocean, the celestial horizon would form the boundary of the visible part of the celestial sphere.

The Apparent Motion of the Stars.—Continued observation shows that, leaving the few planets out of account, the other stars always maintain the same relative positions, and hence they are commonly referred to as the *fixed* stars. Whilst, however, there is no motion relative to one another, they all appear to revolve from East to West in a period slightly less than twenty-four hours round a point in the sky that is known as the *celestial pole*. The motion is just as though the whole celestial sphere, carrying the stars, revolved about an axis passing through this point and its own centre. The ancients, who regarded the earth as a flat plane, thought that this was really what occurred, but we know now that this motion is apparent only, and is due to the fact that we view the stars from a revolving earth. Thus, referring to Fig. 2, the whole of the stars appear to slowly describe circles about a point P in the celestial sphere just as though the whole sphere revolved about the axis O P, so that every star completes its circle in the same time. Some stars, such as A, which are comparatively near to the point P, describe only a small circle, which never takes them below the horizon, so that such stars are always visible. Thus the Southern Cross in the latitude of Southern Australia can be seen at all times, and never sets. Other stars, such as B and C, which are further away from P, describe much larger circles, which take them, as is shown in the figure, below the horizon for a portion of their revolution, so that such stars rise in the East and set in the West. This diurnal motion of the stars may be very prettily demonstrated by exposing a fixed camera containing a highly sensitised plate directed towards the celestial pole on a clear night, leaving the plate exposed for an hour or two. The images of the brightest moving stars will leave trails upon the plate which are all seen to be arcs of circles having a common centre at the celestial pole.

Now, the stars are so distant that their apparent direction in space is absolutely unaltered by any movement of the observer over the earth's surface. The direction of any particular star is precisely the same, even when determined by our most refined instruments, whether viewed from Melbourne, London, or Perth.

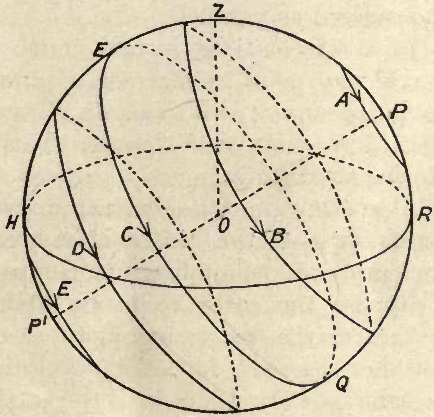


Fig. 2.

More than this, we know that the earth, in the course of a year, describes a path round the sun that is approximately a circle whose diameter is over 190 millions of miles, yet even this great shift of the point of observation produces no appreciable change in the directions of the fixed stars. At intervals of six months apart, when the points of observation, that is to say, are distant something like 190 millions of miles, a slight difference in direction, amounting to only a fraction of a second of arc, may be detected in a few stars with the refined observations possible at fixed observatories. But even this cannot be found with the great majority of the stars, so that we may regard the position of the observer on the earth's surface as of absolutely no importance when measuring the direction of the stars in space. Looking at Fig. 2, we may regard the earth as a tiny speck at O, the centre of the great celestial sphere, and no matter where we take the point O on this tiny speck, the direction of the line O P remains the same within the possibilities of our means of measurement, so that the lines joining any one of the fixed stars to different points on the earth's surface may all be considered as parallel.

It follows from this that the portion of the sky visible to an observer at any point on the earth's surface presents exactly the same appearance as it would do if it were possible for him to view it from the earth's centre. This statement refers only to the fixed stars.

Therefore, if we imagine an observer anywhere on a small spherical earth at the centre of a great celestial sphere of dimensions indefinitely great compared to the earth, and suppose the earth to rotate about an axis through its centre, the successive pictures of the sky presented to the observer during a revolution will be precisely the same as they would be if the earth remained stationary and the great celestial sphere itself were to rotate about the same axis.

Thus, looking at Fig. 2, if we produce the line P O backwards to cut the celestial sphere below the plane of the horizon in P^1, the fixed stars appear to the observer at O to revolve on the celestial sphere about the axis $P P^1$. In reality it is the earth that is revolving, and it is the earth's axis that lies in the direction $P P^1$, so that the celestial poles P and P^1 are the points in which the axis of the earth, if indefinitely produced, would cut the celestial sphere. If the observer is in the Southern Hemisphere, the pole P visible to him will be that to which the earth's South Pole is directed. If he is in the Northern Hemisphere the visible celestial pole is that towards which the earth's North Pole points.

Celestial Equator.—If we take a plane through O perpendicular to the line P P[1], it will cut the celestial sphere in a great circle E Q, which is known as the *Celestial equator*. Its plane clearly is coincident with the plane of the equator of the earth. Since two great circles of a sphere always intersect at opposite extremities of a diameter, it follows that a star revolving in the celestial equator has its path divided into two equal parts by the circle of the celestial horizon H R, so that the time during which it is visible above the horizon will be equal to the time it is out of sight below.

Thus, to an observer in Southern latitudes, the celestial pole P lies to the south and, since the line P P[1] (Fig. 2) marks also the direction of the earth's axis, the celestial pole will be in the direction of the true geographical South. Any star, such as B, lying to the South of the celestial equator, will trace the greater part of its circular path above the plane of the horizon. On the other hand, a star, such as D, to the North of the celestial equator, will trace out the smaller portion of its path only above the horizon, so that it will be visible for less than half of its time of revolution. Stars such as E, sufficiently far to the North, will not be visible at all to a person in this latitude, but will complete the whole of their revolution below the plane of the horizon, as shown in the figure.

Astronomical Co-ordinates—If we wish to mark the position of a point on a plane, we may do so by measuring its distances from two fixed straight lines at right angles. A knowledge of these two distances is sufficient to enable us to fix the position of the point, but one distance only would not be enough. Measured in this way, these two distances are spoken of as the "co-ordinates" of the point. Now, in astronomical observation, we commonly require to determine the position of a star on the celestial sphere, and so it is necessary to have some system of co-ordinate measurement applicable to the purpose. Either one of two sets of co-ordinates is commonly employed. The first set is **Altitude and Azimuth.**

In Fig. 3, let O be the position of the observer, Z the zenith, P the celestial pole. Then the plane Z O P will cut the plane of the horizon through O in the North and South points N and S. S Z N is known as the plane of the *Meridian.*

Suppose that B is a star describing its circular path A B C round the pole P.

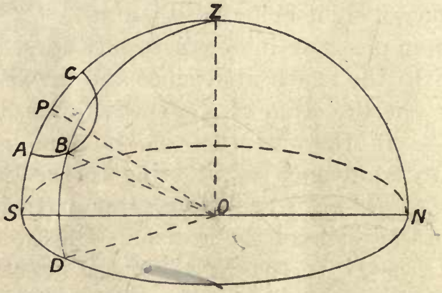


Fig. 3.

The plane Z B O cuts the plane of the horizon in the line D O.

Then it is clear that if we know the angle D O N, which is the angle that the plane Z O D makes with the plane of the meridian, our knowledge is sufficient to fix the position of the plane Z O D.

If in addition we know the angle B O D, the position of the star B may be fixed on the celestial sphere.

The angle D O N, which the plane passing through the zenith and the star makes with the meridian, measures what is known as the *azimuth* of the star. It is generally measured from the North towards the right.

The angle B O D, measuring the angular altitude of the star in a vertical plane above the horizon, is spoken of as the *altitude* of the star.

Instead of the altitude we may measure the angle Z O B, which is known as the *Zenith Distance*, and is clearly the complement of the altitude.

If we know both the altitude and azimuth of a star at any time we can mark its position on the celestial sphere. The ordinary theodolite is adapted for measurement in this system of co-ordinates.

The second or alternative set of co-ordinates is **Right Ascension** and **Declination.**

In Fig. 4, let O be the position of the observer, Z the zenith, P the celestial pole, and S P Z N the plane of the meridian.

Suppose that B is a star travelling round the pole in the direction of the arrow in a circle of which only half is shown.

Q D Q′ is the plane of the celestial equator drawn through O at right angles to O P.

P B D is the arc of a *great* circle of the celestial sphere intersecting the celestial equator in D. The plane of this great circle must pass through O, and the angle P O D is a right angle.

Then clearly if we know the position of the point D on the celestial equator, and also know either the angle P O B or the complementary angle B O D, we shall be able to fix the position of the star B on the celestial sphere.

The position of the point D on the equator may be determined if we know its angular distance from some known fixed point also on the equator. The fixed point selected for the purpose is known as the *First Point of Aries.* It is usually indicated by the symbol ♈, denoting a pair of ram's horns. The exact nature of this point we shall discuss a little later on, but for the present all that we want to know is that it is a point whose position can always be accurately determined.

If we know, then, the angular measure of the arc ♈ D —that is to say, the angle which the arc subtends at the centre O, and also the direction in which it is measured from ♈—that is sufficient to determine D.

To avoid any confusion as to the direction in which the arc ♈ D should be measured, it is always measured from ♈ towards the East—that is to say, in the opposite direction to that in which ♈ travels round the celestial

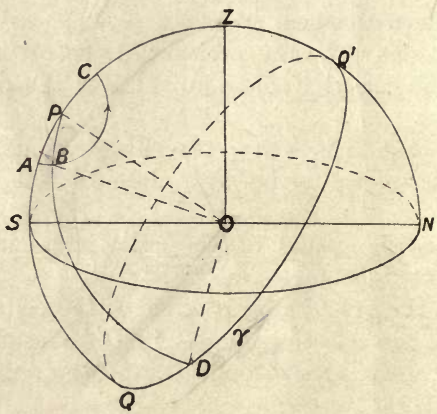


Fig. 4.

equator Q Q′—because ♈ moves round with the rest of the fixed stars from East to West.

Measured in this way, the angular measure of the arc ♈ D is known as the *Right Ascension* of the star B. It may have any value from 0° to 360°. It is commonly denoted by the letters R.A.

The Right Ascension of the star being known, its position may be fixed if we know either the angle P O B, the angular measure of the arc P B, or the angle D O B, the angular measure of the arc D B.

The angular measure of the arc P B is known as the *Polar Distance* of the star B. It is generally denoted by the letters N.P.D. or S.P.D., according as it is measured from the North or the South Pole.

The angular measure of the arc D B is called the *Declination* of the star B, and the circle P B D is known as the *Declination Circle* of the star. The declination is said to be North or South according as the star is North or South of the celestial equator.

Polar Distance and Declination are always complementary to one another, their sum being 90°, so that if one is known the other is found by simple subtraction from 90°.

Comparative Advantages of the two Co-ordinate Systems.—The altitude and azimuth of a star are readily measured with a theodolite, and serve to fix the position of a star at any particular instant, but owing to the diurnal motion of the stars these co-ordinates are continually changing.

On the other hand, the right ascension and declination of a star are constant, for the reference point, the first point of Aries, partakes of the diurnal motion of the stars. These co-ordinates are in consequence the most convenient for recording the relative positions of the stars on the celestial sphere. Thus in the Nautical Almanac the stars are catalogued and tabulated by their right ascensions and declinations.

The Sidereal Day and Sidereal Time.—As the revolution of the whole system of stars about the polar axis takes place with absolute uniformity from East to West, the period of revolution serves as a convenient unit of time for astronomical purposes. All the stars complete their circles of revolution in the same period, which is known as the *sidereal day*. This day is about 4 minutes shorter than the ordinary day. Sidereal clocks, adjusted to keep sidereal time, the sidereal day being divided into 24 hours, are used in fixed observatories. Such clocks are arranged to mark 0 hr. 0 min. 0 sec. when the first point of Aries, the point on the celestial equator from which Right Ascensions are measured, crosses the meridian of the observer. Thus the *sidereal time* at any instant is the interval that has elapsed, measured in sidereal hours, minutes, and seconds, since the last transit across the meridian of the first point of Aries.

Looking at Fig. 4, it is clear that all stars on the same declination circle, such as P B D—that is to say, all stars having the same right ascension—will cross the meridian at the same instant. A star whose right ascension is 180° will cross the meridian 12 sidereal hours after the first point of Aries, and one whose right ascension is 15° will cross the meridian at 1 hr., sidereal time. Thus we deduce the important result that *the right ascension of a star, when reduced to time at the rate of* 24 *hours for* 360° *or* 1 *hour for* 15°, *gives the sidereal time at the moment when it crosses the meridian.*

Hour Angle.—In Fig. 4 the angle ~~B P Z, which is~~ the angle that the plane of the declination circle P B D makes with the plane of the meridian, is known as the *hour angle* of the star B. If we know the hour angle of a star, and also its polar distance, we can clearly mark the position of the star on the celestial globe, so that these two may be used as another system of co-ordinates. The hour angle of a star is continually changing, but owing to the uniform character of the star's motion, it varies at a constant rate. If the hour angle is 90° measured towards the East, then the star will take 6 sidereal hours to reach the meridian. Thus a knowledge of the hour angle at once gives us the time the star will take to reach the meridian, if it be on the East side of it, or the time that has elapsed since the star crossed the meridian, if it be on the Western side.

Prime Vertical.—The plane through the zenith at right angles to the meridian—that is, the vertical plane running

East and West—is known as the *Prime Vertical.* The East and West line, which is the line of intersection of the Prime Vertical with the plane of the horizon, is also the line of intersection of the plane of the celestial equator with the horizon, as will be evident from Fig. 2.

Synopsis of Astronomical Terms.—For purposes of reference, the principal quantities dealt with in this chapter are illustrated in one figure.

Fig. 5*a* is drawn for an observer in the Southern Hemisphere, and Fig. 5*b* for the Northern Hemisphere.

Examples.

1. The R.A. of a star being 35° 20′, what is the local sidereal time when the star is in the meridian?

Ans. 2 hrs. 21 min. 20 sec.

2. If the R.A. of a star is 295° and the sidereal time is 15 hours, is the star to the East or West of the Meridian?

Ans. To the East.

3. What is the declination of a star that rises exactly in the East?

Ans. 0°.

4. What is the hour angle of the star in Question 2?

Ans. 70°.

5. The declination of a star is 35° South. Determine its S.P.D. and its N.P.D.

Ans. 55° and 125°.

6. If the First Point of Aries crosses the meridian exactly two hours, as measured by a sidereal clock, after a certain star, what is the R.A. of the star?

Ans. 330°.

7. The declination of the Pole Star is 88° 51′ North. What is the difference between its greatest and least zenith distances?

Ans. 2° 18′.

8. At the time of the year when the R.A. of the sun is zero, determine approximately the time of rising of a star with declination 0° and R.A. 150°.

Ans. 4 p.m.

9. What is the point whose altitude is 90° and hour angle zero?

Ans. The zenith.

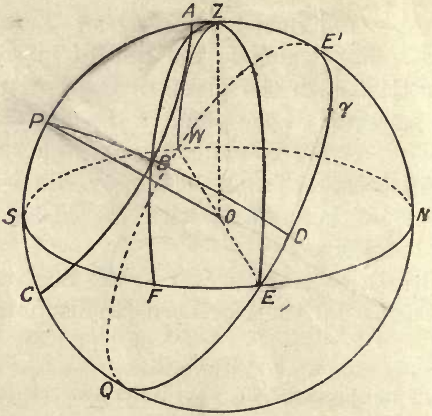


Fig. 5*a*.

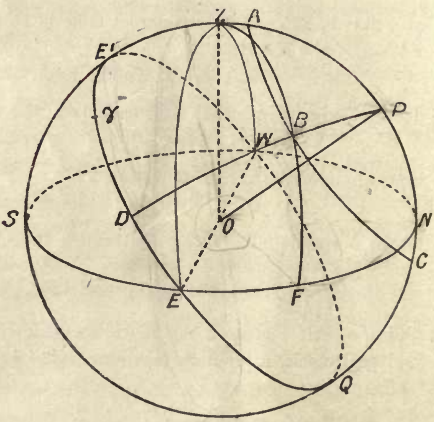


Fig. 5*b*.

O is the observer.
S W N E, the plane of the horizon.
Z, the zenith.
P, the celestial pole ; O P, the polar axis.
S P Z N, the plane of the meridian.
E′ W Q E, the celestial equator.
W Z E, the prime vertical.
N, S, W, E, the North, South, West, and East points.

B, any star.
Z P B, the hour angle of B.
P B D, the declination circle of B.
P B, the polar distance of B.
B D, the declination of B.
♈ D, the right ascension of B.
Z B F, the vertical through B.
B F, the altitude of B.
B Z, the zenith distance of B.
N F, the azimuth of B.

CHAPTER III.

THE EARTH.

The Earth a Globe.—That the earth is a globe is no longer a matter for dispute. It has been circumnavigated and mapped and measured, and no other supposition will fit the facts. We see its round shadow as cast upon the moon during a partial eclipse. We see the planets as great balls of similar dimensions revolving at different distances round the great central sun. The law of gravitation explains the form of their orbits and enables their movements to be predicted with the greatest exactness. That our earth is a globe like these, revolving in a similar way around the sun, is the only satisfactory hypothesis that will account for their apparently involved movements in the heavens. The whole of the apparent movements of the heavenly bodies are readily accounted for on the supposition that the earth is a globe, and no explanation even plausibly satisfactory has been advanced on any other supposition.

In the case of some of the planets we can actually observe that they are in rotation in a manner similar to that in which we assume our own earth must rotate to account for the phenomena of night and day and of the diurnal rotation of the stars. In the planet Mars we see the poles or extremities of the axis of rotation surrounded by white caps apparently similar to the great caps of ice and snow that surround the poles of our own earth.

Terrestrial Latitude and Longitude.—The extremities of the axis of rotation of the earth are called the *Poles*, and are distinguished as the *North* and *South* Poles.

A plane through the earth's centre at right angles to the axis cuts the earth's surface in a circle known as the *Equator*. Every point on the terrestrial equator is thus equidistant from the North and South Poles.

In order to mark the position of a point on the earth's surface, it is necessary to have a system of co-ordinates similar to those we have already discussed in connection with the celestial sphere.

Suppose that P (Fig. 6) is a point on the earth's surface, the position of which it is desired to locate. A plane

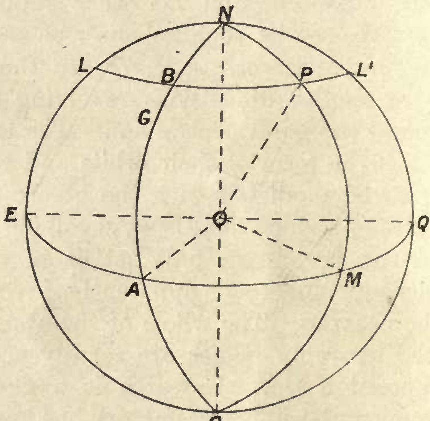


Fig. 6.

passing through P and the earth's axis will cut the earth's surface in a great circle N P M S, which is known as a *Meridian*. Suppose this Meridian cuts the equator at the point M. Then clearly, if we know the position of the point M on the equator, and also the length of the arc P M or the angle which it subtends at the earth's centre, we shall be able to fix the point P.

The position of M on the equator is determined by the *longitude* of P.

To measure this, some arbitrary place A must be selected on the equator as a starting point. The point actually chosen is the point of intersection of the meridian passing through Greenwich, shown as N G A S in the figure, and the equator. The angular measure of the arc A M—that is to say, the angle A O M—is known as the *longitude* of P. Thus, all points on the meridian passing through P have the same longitude. All points on the meridian N G A S, passing through Greenwich, have zero longitude. The longitude of other places is reckoned as so many degrees East or West of Greenwich until we come to 180°, which is the longitude of the meridian exactly opposite to the Greenwich meridian.

The angle P O M, which is the angle between the direction of the vertical at P and the vertical at M, measures what is known as the *latitude* of P. If we draw a plane through P at right angles to the earth's axis, it will intersect the earth in a small circle L P L′ parallel to the equator. Such a circle is called a *Parallel of Latitude*, and all points on the same parallel clearly have the same latitude.

Latitude is measured as so many degrees North or South of the Equator. The latitude of the North Pole is 90° N.

Thus, if we know the position of the meridian of zero longitude, the latitude and longitude of a place are sufficient to enable us to mark its position on the globe.

The Length of a Degree of Longitude.—If the parallel of latitude through P intersects the meridian through Greenwich in B, it is clear that the arc B P will be much smaller than the arc A M. It will have the same angular measurement on a much smaller circle. If P were very near to the North Pole, the arc B P would be very small indeed. Thus two places in the same latitude but differing by, say, ten degrees of longitude, will be very much closer together if they are in a "high" latitude—that is to say, a latitude approaching 90°—than they will be if both are on or near the equator. Thus a degree of longitude has its greatest value, when measured in distance along the earth's surface, at the equator, its value becoming less and less as we approach the poles. At the equator a degree of longitude is equivalent to a distance of about 69 miles.

A degree of latitude, on the other hand, is always of approximately the same value, about 69 miles, whether it is measured near the poles or near the equator, because it is measured along meridians which are all great circles of the same diameter.

The Zones of the Earth.—Certain parallels of latitude

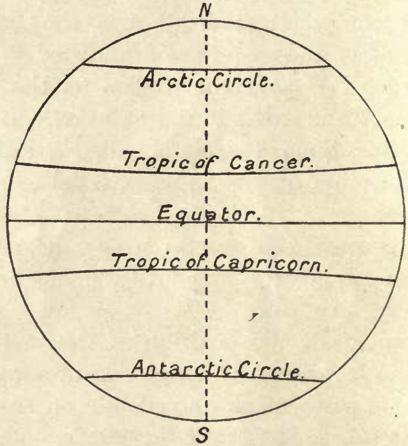


Fig. 7.

divide the earth's surface into five belts or divisions, termed *zones*. These mark in a general way a natural division of the earth's surface according to climate. The parallel of latitude 23° 27½′ North of the Equator is termed *The Tropic of Cancer*, and the corresponding parallel South of the Equator is termed *The Tropic of Capricorn*. As we shall presently see, at all places between these parallels at some part of the year the sun shines directly overhead at mid-day. As a consequence, the belt included between these is the hottest portion of the earth's surface, and it is known as the *Torrid Zone.*

The parallel of latitude 66° 32½′ North of the Equator is called the *Arctic Circle,* and the corresponding parallel South of the Equator the *Antarctic Circle.* The belt between the Arctic Circle and the Tropic of Cancer is known as the *North Temperate Zone,* and that between the Antarctic Circle and the Tropic of Capricorn as the *South Temperate Zone.* The regions around the two poles bounded by the Arctic and Antarctic circles respectively are termed the *Frigid Zones.* At all places within the frigid zones the sun is below the horizon at mid-day for some portion of the year.

The Altitude of the Celestial Pole is Equal to the Latitude of the Place of Observation.—In Fig. 8, let O be the position of the observer and C the earth's centre. Then the direction of the pull of gravity at O is in the direction O C. This, then, will mark the direction of the vertical at O, and the zenith, Z, of the observer will be in C O produced.

H R, at right angles to O Z, marks the plane of this horizon.

If C P, the earth's axis, be produced to cut the celestial sphere in P_1, then P_1 will be the celestial pole.

Draw O P_2 parallel to C P_1.

Then the celestial pole being, as we have seen, at a distance from the earth that is practically infinite in comparison to the earth's radius, O P_2 will mark the direction in which the celestial pole is seen by the observer at O.

Draw the plane of the equator E C Q at right angles to the earth's axis.

Then, from our definition, the latitude of O is measured by the angle E C O.

Now the angle $Z O P_2 =$ the angle $O C P_1$, and the complements of these angles are equal.

Therefore, the angle $P_2 O R =$ the angle $E C O$—*i.e.*, the altitude of the pole = the latitude of the observer.

It follows from this that if the observer travels equal distances North and South from O, since his latitude will change by equal amounts, the altitude of the celestial pole will also be increased or decreased by equal amounts. As this is actually the case from observation, the fact forms a strong proof of the sphericity of the earth.

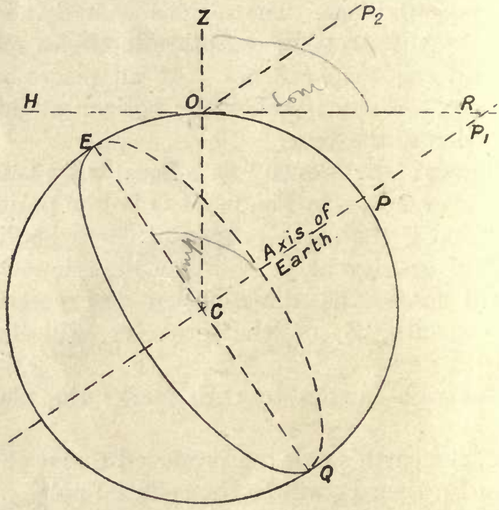


Fig. 8.

To find the Shortest Distance, measured along the Earth's Surface, between two Places whose Latitude and Longitude are given, assuming the Earth to be a True Sphere.

In Fig. 9, let P and R be two places whose latitudes and longitudes are known.

The shortest distance between P and R, measured along the earth's surface, will be the length of the arc of the great circle joining them.

Draw the meridians passing through P and R.

Then if we know the latitudes, we know the angular measure of the meridian arcs N P and N R, N being the North Pole.

If P is in North latitude, the arc N P is the complement of the latitude. If R is the South latitude, the arc N R is 90° + the latitude.

The angle P N R is the difference of the longitudes of P and R if both are measured in the same direction, or

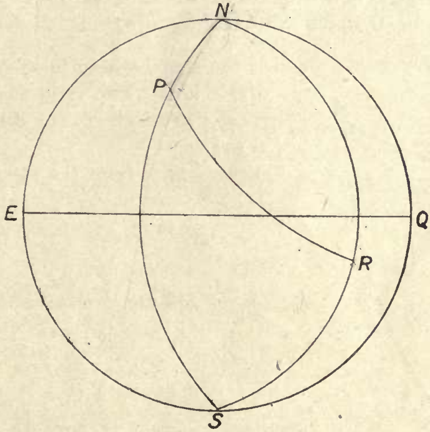


Fig. 9.

the sum of the longitudes, if one is East and the other West.

Thus in the spherical triangle N P R, we know the sides N P and N R and the included angle P N R.

Then by the ordinary methods of spherical trigonometry we can compute the angular measurement of the great circle arc P R, and consequently its lineal measurement, if we know the radius of the earth.

The radius of the earth is approximately 3,960 miles.

EXAMPLE.—*Find the shortest distance measured along the earth's surface between Perth (long.* 115° 50′ *E., lat.* 31° 57′ *S.) and Brisbane (long.* 153° 1′ *E., lat.* 27° 28′ *S.), assuming that the earth is a sphere of radius* 3,960 *miles.*

In this case, both places being in the Southern Hemisphere, it will be preferable to solve the triangle S P R (Fig. 9) rather than N P R.

If A denotes the position of Brisbane, B of Perth, and C the South Pole, we shall have in the spherical triangle A B C

$$CA = b = 90° - 27° 28' = 62° 32'$$
$$CB = a = 90° - 31° 57' = 58° 03'$$
$$C = 153° 1' - 115° 50' = 37° 11'$$

Since we only want to find c, the simplest way to solve this triangle is to divide it into two right-angled triangles by drawing a great circle arc B D to cut C A at right angles.

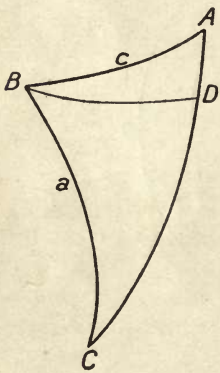


Fig. 9*a*

Then we have from the right-angled triangle B D C

$$\tan CD = \cos C \tan a.$$

$\tan a = \tan 58°\ 3'$, . .	10·2050545
$\cos C = \cos 37° 11'$, . .	9·9012980
$\tan CD$, ...	10·1063525

$\therefore CD = 51° 56' 47''$,

and $\cos c = \cos AD . \cos BD$

$$= \cos(b - CD) . \frac{\cos a}{\cos CD}.$$

$\cos a = \cos 58° 3'$, ...	9·7236026
$\cos(b - CD) = \cos 10° 35' 13''$, . .	9·9925435
	9·7161461
$\cos CD = \cos 51° 56' 47''$, . . .	9·7898616
$\cos c$, ...	9·9262845

$\therefore c = 32° 26' 49''$.

The circular measure of this angle is ·5663.

$\therefore$ The distance required = ·5663 × 3,960 = 2,242·5 miles.

The more usual method of solving the triangle A B C, having given the two sides a, b, and the included angle C, would be to first find the angles A and B by means of the formulæ

$$\tan \tfrac{1}{2}(A+B) = \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \tfrac{1}{2} C,$$

$$\tan \tfrac{1}{2}(A-B) = \frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \cot \tfrac{1}{2} C,$$

and then find c from the formula

$$\sin c = \frac{\sin C \,.\, \sin a}{\sin A}.$$

If this method is adopted to find c, it must be remembered that when $\sin c$ is found there are always two possible solutions, since the sine of an angle = the sine of its supplement. Some care is, therefore, necessary in selecting the appropriate value from the two values determined by the tables.

Examples for Solution.

In all of these examples the earth is to be taken as a sphere of radius 3,960 miles.

1. Find the shortest distance measured along the earth's surface between Mount Gambier (Longitude 140° 45′ E., Latitude 37° 50′ S.) and Palmerston (Longitude 130° 50′ E., Latitude 12° 28′ S.).

Ans. 1,856·8 miles.

2. Find the shortest distance measured along the earth's surface between Baltimore (Lat. 39° 17′ N., Long. 76° 37′ W.) and Cape Town (Lat. 33° 56′ S., Long. 18° 26′ E.).

Ans. 7,893 miles.

3. How far would a place be due South from the equator if the altitude of the S. celestial pole was exactly 20° ?

Ans. 1,382·3 miles.

4. Two places are in S. latitude 30°, one longitude 115° E., and the other 35° E. Find the difference in the paths of the two ships sailing from one port to the other, one along the parallel of latitude and the other along the arc of the great circle joining the places.

Ans. 1,127 miles.

5. What is the declination of a star that passes through the zenith at a place in latitude 35° N. ?

Ans. 35° North.

6. A ship sails along the great circle joining two places, each of latitude 45° N., the difference between their longitudes being $2a$. Show that the highest latitude l reached during the passage is given by the formula $\cot l = \cos a$.

7. A ship from latitude 8° 25′ N. sails south for 600 miles. What latitude is she in?

Ans. 1° 35′ S.

8. At a place in latitude l North, a star with declination d rises 60° E. of North. Show that $\cos l = 2 \sin d$.

The Figure of the Earth.—If, as in Fig. 10, F and G are two points on the same meridian, their difference of latitude will be measured by the angle F O G. If we know this angle, and also the length of the arc F G, we

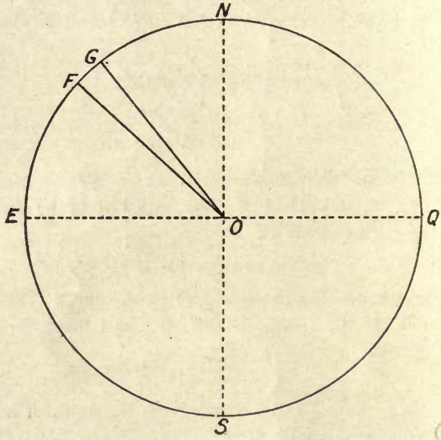


Fig. 10.

shall then be able to calculate the length of the earth's radius F O. The difference of latitude between F and G may be determined by astronomical observation, measuring the altitude of the celestial pole at each place. The length of the arc F G may be either directly measured or it may be computed by means of a triangulation survey from a measured base line on some suitable adjacent part of the earth's surface. Determinations of the radius of the earth on these simple principles were made by the Greeks 2,000 years ago.

If the earth were a true sphere, measurements of the radius of the earth made in this way at different parts of its surface would be all the same. But when it became possible to make the necessary observations with sufficient precision it was found that such was not the case. When Newton discovered and investigated the results of the law of gravitation in the seventeenth century, he proved that one consequence was that if the earth is a plastic body, revolving on an axis and acted on by its own attraction, it must take the form of a slightly flattened sphere with its polar diameter less than its equatorial diameter. Measurements of two arcs made by the Cassinis in France seemed, on the other hand, to indicate that the length of a degree of latitude decreased towards the north, which would imply that the shape of the earth was such that its polar diameter was greater than its equatorial diameter, contrary to Newton's gravitational theory. The French Academy equipped two expeditions in order to settle the problem. One of these measured an arc in the equatorial regions of Peru (1735-1741), and the other an arc in the polar regions of Lapland (1736-1737). The results showed that a degree of latitude was longer in the polar regions than in parts near the equator, and corroborated Newton's theory. Since then many arcs have been measured in different parts of the world, and the observations have conclusively established the fact that the shape of the earth is not a true sphere, but is very approximately an *oblate spheroid*, the figure formed by revolving an ellipse about its minor axis.

The shape of the earth is thus like that of a sphere slightly flattened at the poles. The amount of flattening is not, however, very great. The length of the earth's polar axis may be taken as 7,900 miles, and its equatorial diameter as 7,927 miles. Thus if a model were made 20 feet in diameter, the polar diameter would be shorter than the equatorial by a trifle over three-quarters of an inch.

More exactly still, it is found that the change in the length of a degree of latitude which takes place as we proceed along a meridian is not the same along all meridians. It seems that the equatorial section of the earth is not exactly circular, but is very slightly elliptical. The exact shape would thus appear to be more nearly an *ellipsoid.* For practical purposes, however, all computations in geodetic work are based upon the assumption that the figure of the earth is an oblate spheroid.

Geographical and Geocentric Latitude.—If in Fig. 11 P represents some point on the meridian N Q S, N and

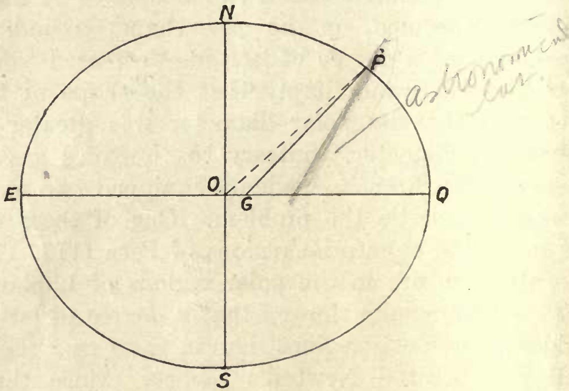


Fig. 11.

S being the North and South Poles, then, making allowance for the fact that the section N Q S E is not a circle but an ellipse, the direction of the horizontal at P will not be at right angles to P O, O being the earth's centre, but will be in the direction of the tangent to the ellipse at P. This is the direction taken by the surface of still water at that point. Consequently the direction of the vertical there is not O P but G P, where G P is the normal at P—that is to say, it is at right angles to the tangent. Thus, if we measure the latitude of P by astronomical methods, observing the altitude of the celestial pole above the horizon at P, we shall measure the angle P G Q and not the angle P O Q. The angle P G Q thus measures what is called the *geographical* or *geodetic latitude.* This is the ordinary latitude that is used for astronomical and geodetic purposes.

It is clear, however, that the value of the angle P O Q, if it can be readily determined, might be equally well used in order to fix the position of P on the meridian. This angle measures what is termed the *geocentric latitude*. The difference between the geocentric and geographical latitude of a place is never very great. There is no difference at all, either at the poles or at the Equator, and the maximum difference is in latitude 45°, where it amounts to about 11′ 44″ of arc. The geocentric latitude cannot be directly observed. It is computed from the geodetic latitude by the formula :—

$$\tan P\,O\,Q = \frac{O\,N^2}{O\,Q^2} \tan P\,G\,Q.$$

When speaking of latitude in this book, it will always be the geodetic latitude that is meant unless otherwise specified.

Examples.

1. At a place in Lat. 42° S. a line is run from a point A on a bearing of 220° for a distance of 2,400 chains to a point B.

Assuming the earth a sphere of 3,957 miles radius, find the bearing from B to A.

Ans. 40° 15′ 12″.

2. Given that latitude of London is 51° 32′ N., latitude of Jerusalem 32° 44′ N., bearing of Jerusalem from London, 110° 04′. Find the longitude of Jerusalem, its distance from London, and the bearing of London from Jerusalem.

Ans. Longitude, 37° 25′ 12″ E.
Distance, 2,278 miles.
Bearing of London from Jerusalem, 316° 00′ 16″.

3. The latitude of a Trig. Station A is 33° 51′ S., and its longitude is 151° 12′ 42″ E. The bearing and distance to another Trig. Station B is 284° 08′ 44″, 105,600 feet.

Compute the latitude and longitude of B, and the bearing of B to A, on the assumption that the earth is a sphere with radius 20,890,790 feet.

Ans. Longitude, 151° 28′ 48″ E.
Bearing, 104° 56′ 20″.

4. Find the great-circle distance in English statute miles from Wellington, N.Z., to Panama, treating the earth as a sphere, and one degree as equal to $69\frac{1}{20}$ statute miles.

Wellington, . .	Lat. 41° 17′ S.,	Long. 174° 47′ E.
Panama, . .	Lat. 9° 00′ N.,	Long. 70° 31′ W.

Ans. 4,528·6 miles.

5. Two places are each in latitude 50° N., and their difference of longitude is 47° 36′. Find their distance apart.

Ans. 2,090 miles.

CHAPTER IV.

THE SUN.

The Sun's Apparent Motion among the Stars.—Like the fixed stars, the sun shares in the apparent general daily rotation of the heavens, but unlike them it does not always maintain the same position relative to other objects on the celestial sphere. In addition to its daily circling of the sky, it appears to gradually shift its position with respect to the stars. Neither its declination nor its right ascension remains constant. Very little consideration will show that its declination must alter during the year, for, if it did not, the sun would always describe the same circle in the heavens. If this were the case, then, like the fixed stars, it would always rise and set at the same points on the horizon, and it would always attain the same altitude when on the meridian. Since it does not do this, it is clear that the declination of the sun must change during the year. That the sun has also a movement in right ascension among the stars is not quite so obvious, but the fact may be readily inferred if we watch the stars that are visible in the East on succeeding mornings just before sunrise or in the West just after sunset. Stars in the East that rise just before the sun, so that in a very short time after rising they are masked by the sun's rays, will on each succeeding morning be seen for a longer time. Similarly stars in the West, setting just after the sun, will be visible for shorter and shorter periods as we watch them on successive evenings until finally they are lost altogether in the strong sunlight, other stars further East taking their places. Hence we infer that the sun has a progressive movement among the stars from West to East.

The problem of determining the sun's place on the celestial sphere with regard to the fixed stars was a difficult one to early astronomers, because as soon as the sun becomes visible its strong light prevents the stars from being observed at the same time. Some used the moon, and Tycho Brahe used the bright planet Venus in order to get the connection, observing the relative position of the sun and moon or of the sun and Venus when both were visible, and afterwards measuring the position of the moon or Venus with regard to the stars when the sun had set. But as both the moon and Venus also move amongst the stars, the movement that had taken place in the interval had to be allowed for, and the method was thus not particularly simple. The sun's position is nowadays determined by much more accurate methods.

The Earth's Orbit round the Sun.—All of these movements of the sun are apparent only and not real. Just as its apparent daily rotation in the heavens is due to the rotation of the earth on its axis, so the sun's apparent movements in right ascension and declination are really due to the fact that the earth moves in a great orbit round the sun once a year.

Actually the earth moves round the sun in a path that is very nearly a huge circle with a radius of about 96 millions of miles. More accurately, the path is described as an ellipse, one focus of the ellipse being occupied by the sun. The curve traced out by the centre of the earth lies in a fixed plane that passes through the centre of the sun. The earth traces out its complete orbit once a year, and all the time it is spinning on its own axis once a day, the direction of the spin on its axis being the same as that in which it moves round the sun. The earth's axis is not at right angles to the plane of its orbit, but it makes with the plane a fixed invariable angle of $66° \ 32\frac{1}{2}'$. That the direction of the earth's axis is constant we know from the fact that the position of the celestial pole amongst the fixed stars shows no appreciable shift throughout the year. Thus, as is illustrated in Fig. 12, the earth moves round the sun, spinning on its axis, which is inclined to the plane of the orbit, and the axis always remains parallel to itself, pointing ever in the same direction amongst the fixed stars, whose distances, it must be remembered, are practically infinitely great even in com-

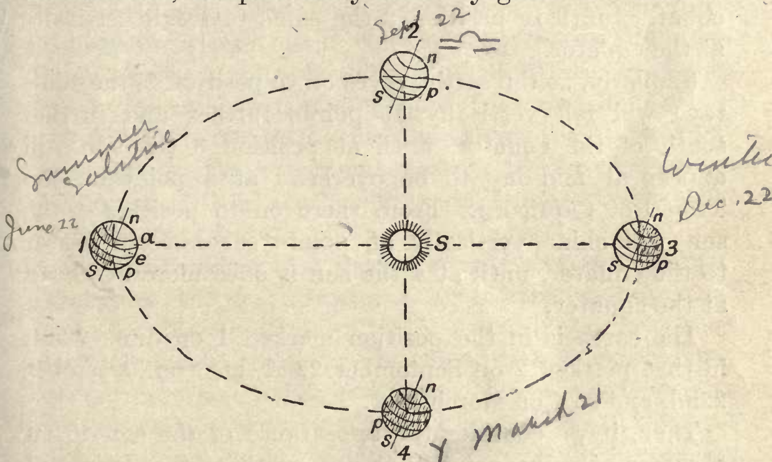


Fig. 12.

parison with the immense distance of the earth from the sun.

When the earth is in the position marked 1, the sun will be shining directly overhead in a place such as *a* North of the equator. If *e* is a point on the earth's equator on the same meridian of longitude as *a*, O being the earth's centre, the angle *a* O *e* will be the complement of $66^\circ\ 32\frac{1}{2}'$ or $23^\circ\ 27\frac{1}{2}'$—that is to say, *a* will be a point on the Tropic of Cancer. In this position, then, the sun at mid-day will be vertically overhead at all points on the Tropic of Cancer. This statement is not quite accurate, because the earth does not remain in the one position in its orbit while it makes a complete revolution on its axis; it is moving forward in its orbit all the time, but as it takes a whole year to go round the sun, its relative movement is not very great in one day.

As the earth moves from position 1 to position 2, its axis always remaining parallel to its original direction, it will be seen that the sun will appear to shine directly overhead at points successively nearer and nearer to the equator, until in position 2 the sun's rays fall vertically at the equator.

Similarly, as the earth moves on to position 3, the sun's rays will fall vertically at points further and further south of the equator, until at position 3 the sun will appear at mid-day to be overhead at a point on the Tropic of Capricorn. From there on to position 4 the sun will shine vertically at points successively nearer to the equator, until at 4 the sun is once more overhead at the equator.

The earth is in the position marked 1 on June 22nd, in that marked 2 on September 22nd, at 3 on December 22nd, and at 4 on March 21st.

Thus, if we consider the appearance of the sun to an observer at some point P to the south of the Tropic of Capricorn, on June 22nd the sun will appear to be further from the zenith and lower down in the sky than at any other period of the year. On December 22nd, when the earth is in position 3, the sun at mid-day will be nearer the zenith than at any other time of the year.

The orbit of the earth being an ellipse, its distance from the sun is not constant. It is furthest from the sun in the position 1, and nearest to the sun in the position 3.

The Equinoxes.—On March 21st and September 22nd, the sun, being vertically overhead at the equator, will appear to an observer at any part of the earth to be in the celestial equator. Now, we have seen that when any heavenly body is in the celestial equator its path is bisected by the horizon, so that the time during which it can be seen in the sky is equal to the time during which it is invisible. Thus, when the earth is in either of these positions the days and nights are of equal length all over the world. These points are consequently called the *Equinoxes.*

Motion in Right Ascension and Declination.—It thus appears that on March 21st and September 22nd the sun's declination is zero, as it lies on the Celestial Equator. From March 21st to September 22nd it will appear in the sky to the North of the equator, so that its declination will be north with a maximum value of $23° \ 27\frac{1}{2}'$ on June 22nd. From September 22nd to March 21st its declination will be south with a similar maximum value on December 22nd.

It is also evident that the sun's right ascension changes throughout the year, because as the earth revolves round it the apparent position of the sun among the fixed stars must obviously change. The stars that would be seen by an observer on the earth when in position 1, looking in the direction of the sun, would be seen by an observer at 3 when looking in the direction opposite to that of the sun. Clearly, in the course of the year the sun will trace out a complete circle among the fixed stars.

The declination and right ascension of the sun are given in the Nautical Almanac for Greenwich noon on every day of the year. The values at intermediate instants may be found by interpolation. Illustrations of such calculations are given in Chapter VIII. when dealing with sun observations.

The Sun's Semi-Diameter.—The disc of the sun subtends at the eye of an observer an angle of about half a degree. By accurately measuring the angle subtended by diameters taken in different directions, we find that these are all equal, so that the disc is circular in form. In order to mark the position that the sun occupies on the celestial sphere at any time, we require to determine the position of the centre of the circular disc. But there is no mark at the centre that we can recognise, and so in practice we must observe a point on the edge of the sun and then make an allowance for the distance of this point from the sun's centre.

From what we have just seen of the nature of the earth's motion round the sun, it is clear that the sun is not at all times of the year at the same distance from us, and consequently we should not expect its diameter to remain constant. As the earth completes its orbit round the sun in a year and then goes over the same path again, we might anticipate that the variations in the value of the sun's apparent diameter would follow a yearly cycle. This is found to be the case, a slow decrease taking place from the 31st of December to the 3rd of July, and a slow increase during the second half of the year.

As the semi-diameter is frequently required in reducing sun observations, the values are chronicled for every day in the year in the Nautical Almanac (p. 11 of each month). In the almanac for 1914 the maximum value of the semi-diameter is given on January 3rd as $16' \ 17{\cdot}55''$, and the minimum on July 3rd as $15' \ 45{\cdot}38''$.

To Plot the Position of the Sun's Centre on the Celestial Sphere.—Supposing that we know the direction of the true North and South, and also the latitude of the place of observation, we may readily measure the declination of the sun at mid-day. With a telescope pointed in the direction of the meridian we may observe the altitude of the sun's upper or lower edges (*limbs*, as they are usually called) at the moment when it crosses the meridian. Making due allowance for the sun's semi-diameter, we shall thus obtain the meridian altitude of the sun's centre.

Thus, as in Fig. 13, if P represents the Pole, Z the zenith, we measure either S_1 N or S_2 S, according as the sun is in a position such as S_1 or as S_2. Now, we have previously shown that the altitude of the celestial pole, P N, is equal to the latitude of the place. Thus, if the sun is situated as at S_1, on the same side of the zenith as the pole, the difference between the observed altitude S_1 N and the latitude P N gives the sun's polar distance P S_1. If the sun is at S_2, on the opposite side of the zenith to the pole, then the arc S_2 N is equal to 180° — the observed altitude S S_2. The difference between S_2 N and the latitude P N gives the sun's polar distance as before. The declination of the sun is the complement of its polar distance.

180°

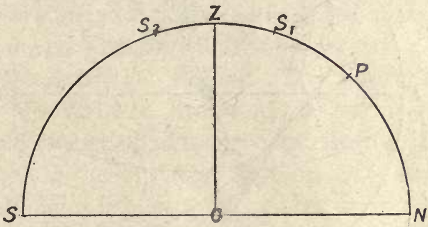


Fig. 13.

Having measured the declination of the sun in this way, in order to fix its position on the celestial sphere, it only remains to determine the difference between its right ascension and that of some star whose co-ordinates are known. But we have seen that the difference of right ascension of any two stars is measured by the interval in time between their transits across the meridian, as given by the sidereal clock. If, with the sidereal clock, the times be measured when the first and second limbs of the sun cross the meridian, the mean of the two times will give the instant when the centre crosses the meridian. If, therefore, the time of passage across the meridian of some selected known star is also observed, the interval between the two times, reduced to degrees, will give the difference between the right ascension of the sun and the star.

These observations give us the elements necessary to plot the position of the sun.

The Sun's Apparent Annual Path on the Celestial Sphere.—In Fig. 14, let A represent the position of the selected fixed reference star as plotted on a globe representing the celestial sphere, P being the Pole, Q R the great circle of the equator, and S N the horizon. Then, if we

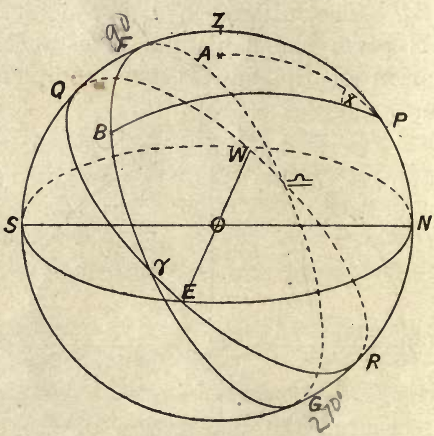


Fig. 14.

set out the angle A P B equal to the observed difference of right ascension and measure off the arc P B equal to the observed polar distance of the sun, the point B will represent the position of the sun's centre on the star globe.

When observations similar to those just described are made day after day, and the corresponding positions of the sun plotted on the globe, those positions are all found to lie on a great circle, which cuts the equator at two opposite points ♈ and ♎ in the figure, and is inclined to it at an angle of about 23° 27′.

* therefore the pole of the ecliptic is a line perpendicular to the sun's yearly path, passing through its centre.

The great circle, the plane of which contains the sun's yearly path, is called the *ecliptic*, and the angle this makes with the equator is spoken of as the *obliquity of the ecliptic*.

Its points of intersection with the equator are called the *equinoctial points*, one (♈) is known as the *First Point of Aries*, and the other (♎) as the *First Point of Libra*.

The sun is at the first of these points on about the 21st of March (the *vernal equinox*), and at the second on the 23rd of September (the *autumnal equinox*), its declination being then 0° and its polar distance 90°.

As we have already seen, ♈ is the point selected on the equator as that from which right ascensions are measured, so that the right ascension of ♈ is 0° and that of ♎ 180°.

At the two points on the ecliptic whose right ascensions are respectively 90° and 270°, the sun will have its greatest declination north and south of the equator. These are known as the *Solstitial Points.* The sun reaches them on or about the 22nd of June and the 22nd of December. On June 22nd the sun has its greatest declination of about 23° 27′ north of the equator, and on December 22nd its greatest declination south.

Examples.

1. Determine the meridian altitude of the sun at a place in latitude 30°, (*a*) at the equinoxes, (*b*) during the summer solstice.

Ans. 60° and 83° 27′.

2. Find the latitude of the place where the greatest altitude of the sun in midsummer is 60°.

Ans. 53° 27′.

3. At a place in lat. 80° N., on a certain day the sun at mid-day just appears above the horizon. Find the sun's declination. Find also the altitude of the sun at mid-day when its declination is 20° N.

Ans. 10° S. and 30°.

CHAPTER V.

TIME.

Sidereal Time.—To measure time we require some form of perfectly uniform motion, and the most perfect motion of this kind in the heavens is provided by the apparent revolutions of the fixed stars. The earth turns on its axis with absolutely regular speed and, as the stars are so distant that the movement of the earth in its orbit round the sun produces no apparent effect upon their relative positions, the consequence is that the stars complete a revolution round the celestial pole at a perfectly regular rate in a fixed and constant time. To the astronomer, then, this presents the simplest way of measuring time. The period of a complete revolution of the stars round the pole is known as the sidereal day, and time measured in this way is termed sidereal time.

Apparent Solar Time.—Convenient as the above method of measuring time is to the astronomer, it is obviously unsuited to ordinary purposes of life. It is the day as determined by the sun that controls our habits and rules our lives. The *apparent solar day,* or period of time between successive transits of the sun across the meridian, is, however, variable in length, and it is impossible to regulate a clock so that it shall indicate exactly 12 o'clock just when the sun is in the meridian. The reason of this may be seen from Fig. 15, which shows in an exaggerated way the movement of the earth in its orbital revolution round the sun. Suppose that, when the earth is in the position marked 1, the sun is directly overhead to an observer at A, and that, if it could be seen, the star F would appear in the same direction. As the earth revolves on its axis it also travels forward in its orbit, so that at the end of a sidereal day it is in the position marked 2. If the observer has been carried round to the point B, so that the same star F appears vertically overhead, the star being at practically an infinite distance, B F will be parallel to A F. The interval between these two positions marks a sidereal day. But to bring the sun overhead, to the same observer, he must wait till he is carried round the extra distance B C. The solar day then will be longer

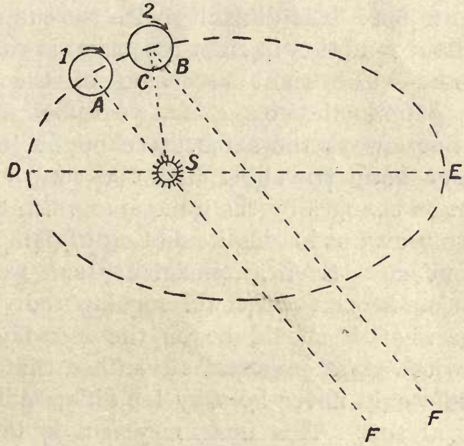


Fig. 15.

than the sidereal day by the length of time required to traverse this extra distance. Whilst the sidereal day is the time taken by the earth to make a complete revolution on its axis, the apparent solar day is the time taken to make a little more than a revolution.

Now, the earth does not move in a circular but in an elliptic orbit round the sun, so that sometimes it is nearer to the sun than at others. When it is nearer to the sun it is a deduction from the law of gravitation that it must travel faster in its path than when it is further away. The result is that the extra little bit, B C, through which the earth has to turn in the interval of time that has to be added on to the sidereal day to give the apparent solar day, is not always the same, and the apparent solar day is thus not of constant length.

We have seen that the right ascensions of the fixed stars are practically constant. But if a celestial body were to move in right ascension its period of revolution about the pole would still be constant, although not the same as that of the stars, provided the movement was a uniform one. The difficulty with the sun as a time-keeper is that its motion in right ascension is variable.

Mean Time.—The right ascension of the real sun changes by 360° in the course of a year, but the rate of change is not always the same. We might conceive of an imaginary body travelling with the sun, so that its right ascension changes by the same amount in the course of the whole year, but having its motion in right ascension perfectly uniform. Such an imaginary sun would form a perfect time-keeper, we could regulate our clocks to mark noon when it should be on the meridian, and it would have the great practical advantage that the time so indicated would never be very far different from that of the actual sun. This imaginary sun is termed the *mean sun*, and the time indicated by it is called *mean solar time*. The mean sun is pictured as moving along the equator with uniform speed, so that its motion is the average of that of the actual sun in right ascension. A *mean solar day* is the interval between two successive transits of the mean sun across the meridian.

The Three Systems of Time Measurements.—There are thus three kinds of time to be considered.

1. *Sidereal*, as determined by the revolution of the stars.
2. *Apparent solar*, as measured by the actual sun or a sun dial.

3. *Mean solar*, which is the ordinary time kept by our clocks.

The hour angle (Chap. II.) of the real sun gives the apparent time or time indicated by a sun dial, and the hour angle of the mean sun gives the mean time at that instant.

Mean noon is the instant when the mean sun is on the meridian. The *mean time* at any other instant is measured by the hour angle of the mean sun reckoned westward from 0 hr. to 24 hrs. Thus the astronomical mean day is usually divided into 24 hours instead of the two divisions of 12 hours each in common use for civil purposes. As the astronomical day starts at noon, both methods will agree in the afternoon of each day, but not in the morning. Thus, July 29th, 10 p.m., would be the same in both the civil and astronomical methods of reckoning, but July 29th, 10 a.m., Civil time, would be equivalent to July 28th, 22 hrs., astronomical time.

Equation of Time.—The difference between the mean and the apparent solar time is known as *The Equation of Time.* It is counted positive when the mean time exceeds the apparent time, and negative when the apparent time is greater than the mean. It is thus always the amount that must be added to the apparent to obtain the mean time. Thus we have—

Mean Time = *apparent Time* + *Equation of Time.*
or *Clock Time* = *sun dial Time* + *Equation of Time.*

When the actual sun is on the meridian, the sun dial will indicate 0 hr. or noon. Hence—

Equation of Time = *mean Time of apparent noon.*

The equation of time is thus positive if the sun is "after the clock," or the true sun transits after the mean sun. Its values at both mean and apparent noon at Greenwich are tabulated in the Nautical Almanac for every day in the year.

The equation of time vanishes four times a year, on or about April 15th, June 15th, September 1st, and December 24th. From December 24th till April 15th it is positive, with a maximum value of about 14 min. 26 sec. on February 11th. From April 15th to June 15th it is negative, having its greatest value of about 3 min. 48 sec. on May 15th. From June 15th to September 1st it is again positive with a maximum value of about 6 min. 19 sec. on July 27th. Between September 1st and December 24th it is negative once more, attaining its greatest negative value for the year, about 16 min. 21 sec. on November 3rd. These dates are approximate only, as they are not always precisely the same in different years.

It will be seen on looking at the tabulated values of the equation of time in the Nautical Almanac, that it is a continuously varying quantity, its value commonly changing by several seconds from one day to the next. The tabulated values are for Greenwich noon, and consequently if we wish to know the equation of time at some other instant we must find its value by interpolation. To facilitate this the Nautical Almanac gives the value of the variation in one hour at each noon.

For example, the equation of time at Greenwich mean noon on March 21st, 1913, is given as 7 min. 25·89 sec., and is diminishing from day to day. The variation in one hour at noon on March 21st is 0·755 second. If, then, we require the equation of time at 11 hrs. on March 21st (Greenwich time), all we have to do is to subtract $11 \times 0{\cdot}755$ sec. from 7 min. 25·89 sec., giving, as the equation of time at the required instant, 7 min. 17·59 sec.

If it is desired to make the computation with the greatest precision, allowance must be made for the fact that the rate of variation given is the rate at Greenwich noon, and not the mean rate over the 11 hours. The rate of variation at noon on the next day, March 22nd, is given as 0·760 sec., and, therefore, the rate of variation $5\frac{1}{2}$ hours after noon on March 21st is $0{\cdot}755 + \frac{5\frac{1}{2}}{24} \times 0{\cdot}005 = 0{\cdot}756$. This would more accurately represent the mean rate of variation during the 11 hours, and the required equation of time is, therefore, more accurately, 7 min. 25·89 sec. $-$ 11 $\times$ 0·765 $=$ 17 min. 17·57 sec.

The more accurate procedure thus only makes a difference in the second place of decimals of a second, and the simpler method given at first is good enough for most purposes.

EXAMPLE.—Find the equation of time at 5 hrs. 30 min. on February 25th, the equation of time at noon being 13 min. 17·86 sec. and the variation in one hour 0·395 sec.

Ans. 13 min. 15·69 sec.

Local Mean Time—The *local mean time* at any place is reckoned by counting as 0 hr. the instant when the mean sun last crossed the meridian of the place. As the earth rotates uniformly on its axis from West to East, it follows that the further East a place is situated the sooner will the sun cross the meridian, and, therefore, the *later* will be the local time. All places on the same meridian of longitude have their noons at the same instant, and, as the earth turns, one meridian after another is brought opposite to the sun. Thus, the interval of time between the local noons at two different places will depend upon their difference of longitude.

As the earth turns through 360° in 24 hours, it follows that a difference of 15° of longitude corresponds to a difference of 1 hour in time, 15′ of arc corresponds to a difference of 1 minute of time, and 15″ of arc to a difference of 1 second of time.

Thus, if we know the longitude and the local time at one place A, we can readily compute the time at any other place B whose longitude is given. We have only

4

to convert the difference of longitude into time, at the rate of 15° per hour, and add this to the time at A if B is to the East, or subtract it if B is to the West from A.

EXAMPLE.—If the longitude of A is 36° 03′ 37″ E., and the local mean time is September 5th 1 hr. 31 min. 17 sec., find the time at B in longitude 3° 27′ 13″ E.

The difference of longitude = 32° 36′ 24″.

To convert this into time, we simply have to divide by 15, giving us, as the difference in time between the two places, 2 hrs. 10 min. 25·6 sec.

As B is to the West from A, this has to be subtracted from 1 hr. 31 min. 17 sec., giving us as the time at B, September 4th, 23 hrs. 20 min. 51·4 sec.

Should one longitude be East from Greenwich and the other West, we must add them, instead of subtracting, in order to get the angle between the meridians.

EXAMPLE.—A ship sails from London on January 2nd at 1 p.m., and arrives in Melbourne (longitude 145° E.) at 6 p.m. on February 8th. Find the time occupied by the voyage.

Ans. 36 days 19 hrs. 20 min.

Local Sidereal Time.—The local sidereal time at any place is reckoned by counting as 0 hr. the instant when the First Point of Aries last crossed the meridian of the place. Therefore, in precisely the same way, if we know the longitudes of two places A and B and the *local sidereal time* at A, we can compute the corresponding sidereal time at B. For the earth turns on its axis through 360° relative to the fixed stars in 24 sidereal hours, and, therefore, a difference of longitude of 15° corresponds to a difference of 1 hr. in the sidereal times. The method to be used for finding the sidereal time at B is thus exactly the same as that just illustrated.

EXAMPLE.—If the sidereal time at A, long. 35° E is 12 hrs. 30 min., find the sidereal time at the same instant at B, long. 27° W.

Ans. 8 hrs. 22 min.

Apparent Solar Times at the Same Instant at Places in Different Longitudes.—The equation of time or difference between apparent and mean times is the same all over the world at the same instant. Consequently the difference between the apparent solar times at two places A and B is precisely the same as the difference between the local mean times. The same method again then can be used to determine the apparent time at B, having given the apparent time at A.

EXAMPLE.—If the apparent solar time at A, long. 45° W. is 1 hr. 30 min., and the equation of time is 6 min. 10 sec., to be added to apparent time, find the corresponding mean time at B in longitude 10° W.

Ans. 3 hrs. 56 min. 10 sec.

Standard Time.—To avoid the confusion arising from the use of different local times in each town, most countries now adopt the system of using the time on a particular meridian through the country that lies an even number of hours from Greenwich. The following table shows the standard times adopted by the principal countries of the world :—

Longitude of Standard Meridan.		Countries.
In Degrees.	In Time. Hrs. Min.	
172½° E.	11 30 E.	New Zealand.
150° E.	10 0 E.	Victoria, New South Wales, Queensland, Tasmania.
142½° E.	9 30 E.	South Australia.
135° E.	9 0 E.	Japan, Corea.
120° E.	8 0 E.	Western Australia.
82½° E.	5 30 E.	India.
30° E.	2 0 E.	East Europe, South Africa, Egypt.
15° E.	1 0 E.	Germany, Austria, Denmark, Sweden, Norway, Switzerland, Italy, Western Turkey.
0°	0 0	Great Britain, Belgium, Spain.
60° W.	4 0 W.	Atlantic Provinces of Canada.
75° W.	5 0 W.	Quebec, Eastern Zone of the United States, Peru.
90° W.	6 0 W.	Central Zones of Canada and U.S.A.
105° W.	7 0 W.	Mountain Zones of Canada and U.S.A.
120° W.	8 0 W.	British Columbia and the Pacific Zone of U.S.A.

To Change Standard Time to Local Mean Time.—This problem has really been already discussed, for the difference between standard time and local mean time at any place is that due to the difference of longitude between the given place and the standard time meridian used. For places East of the standard meridian local mean time is later than standard time, and for places to the West the local time is earlier.

EXAMPLES.

The standard time meridian in South Australia being 142° 30′ E., find the local mean time at Adelaide (longitude 138° 35′ E.) when the standard time is 8 hrs. 25 min. 10 sec.

Ans. 8 hrs. 9 min. 30 sec.

In New York State the standard time meridian is 75° W. If the local mean time is 10 hrs. 17 min. 18 sec. at a place in the State, the longitude of which is 73° 58′ W., find the standard time.

Ans. 10 hrs. 13 min. 10 sec.

To Reduce a Given Interval of Mean Time to Sidereal Time and *vice versa.*—It will be seen from the consideration of Fig. 15 that in the course of its complete orbital revolution round the sun the earth will make exactly one turn less with respect to the sun than it does with respect to the fixed stars. There are approximately $365\frac{1}{4}$ mean solar days in the year, and, therefore, in the same period there are $366\frac{1}{4}$ sidereal days. More exactly, according to Bessel, the year contains 365·24222 solar days, and hence 365·24222 solar days = 366·24222 sidereal days.

Therefore, if m be the measure of any interval in mean time and s the corresponding measure in sidereal time,

$$\frac{m}{s} = \frac{365{\cdot}24222}{366{\cdot}24222}.$$

Thus, if m be given, s can be found, or *vice versa.*

Tables to facilitate the reduction are given in the

Nautical Almanac, and less elaborate ones in Chambers' Mathematical Tables.

When tables are not used, the simplest way to make the computation is as follows :—

To convert an interval of mean solar time to sidereal time, *add* 9·8565 seconds for each mean solar hour. Dividing by 60, this gives us ·1642 second to be added for each minute and ·0027 second for each second of mean time.

Thus, to convert an interval of 6 hrs. 33 min. 17 sec. of solar time into the equivalent interval of sidereal time, we have—

$$\begin{array}{rrr} 6\times & 9{\cdot}8565 = & 59{\cdot}139 \\ 33\times & {\cdot}1642 = & 5{\cdot}418 \\ 17\times & {\cdot}0027 = & {\cdot}046 \\ \hline & & 64{\cdot}603 \end{array}$$

64·603 seconds = 1 min. 4·6 sec.

The addition of this to the given solar time gives us 6 hrs. 34 min. 21·6 sec. as the equivalent sidereal interval.

To convert an interval of sidereal time to the equivalent interval of mean solar time, *subtract* 9·8296 seconds for each sidereal hour. Dividing by 60 we get ·1638 second to be subtracted for each sidereal minute, or ·0027 second for each second.

Thus, to find the interval of solar time equivalent to an interval of 6 hrs. 33 min. 17 sec. of sidereal time, we have—

$$\begin{array}{rrr} 6\times & 9{\cdot}8296 = & 58{\cdot}978 \\ 33\times & {\cdot}1638 = & 5{\cdot}405 \\ 17\times & {\cdot}0027 = & {\cdot}046 \\ \hline & & 64{\cdot}429 \end{array}$$

64·429 seconds = 1 min. 4·43 sec.

Subtracting this from the given interval of sidereal time gives 6 hrs. 32 min. 12·57 sec. as the equivalent mean time interval.

Given the Sidereal Time at Mean Noon at Greenwich on any given Date to find the Local Sidereal Time at Local Mean Noon at any other Place on the Same Date.

On page 11 for each month in the Nautical Almanac the Greenwich sidereal times are tabulated for Greenwich mean noon on each day. From these it is necessary, in most work in which the time has to be brought into the calculations, that we should be able to deduce the local sidereal time at local mean noon on the corresponding day at the place of observation.

In the succeeding pages it will be convenient to use the following abbreviations :—

G.M.T.	to denote	Greenwich mean Time.
G.S.T.	,,	Greenwich sidereal Time.
G.M.N.	,,	Greenwich mean noon.
L.M.T.	,,	Local mean Time.
L.S.T.	,,	Local sidereal Time.
L.M.N.	,,	Local mean noon.

From what we have already done, it will be evident that if we have two clocks, one set to keep sidereal time and the other to keep mean time, the sidereal clock will complete its day in a shorter period than the other, and consequently will be continually gaining. According to the last article, it will gain at the rate of 9·8565 seconds for each solar hour.

Now, at a place in West Longitude, noon occurs a certain number of hours *after* noon at Greenwich, the interval depending upon the longitude. But the tabulated sidereal time at Greenwich noon is the difference between the readings of the sidereal and mean time clocks at that instant. Consequently, by the time it becomes noon at the place in question, the sidereal time will have gained still further on the mean time clock at the rate of 9·8565 seconds for each hour of longitude. Thus the L.S.T. at L.M.N. will be *greater* than the G.S.T. at G.M.N. by an amount computed at the rate of 9·8565 seconds for each hour of West longitude.

Similarly, at a place in East Longitude, noon occurs before the corresponding noon at Greenwich, and in this case L.S.T. at L.M.N. will be less than the G.S.T. at G.M.N. by an amount computed in the same way according to the longitude.

Example.—On October 1st, 1914, the G.S.T. at G.M.N. is given in the Nautical Almanac as 12 hrs. 37 min. 29·99 sec. Determine the L.S.T. at L.M.N. (*a*) at a place in longitude 57° 33′ 28″ West, (*b*) at a place in the same longitude East.

(*a*) 57° 33′ 28″ is equivalent to 3 hrs. 50 min. 13·87 sec.

$$\begin{aligned} 3 \times 9{\cdot}8565 &= 29{\cdot}569 \\ 50 \times 0{\cdot}1642 &= 8{\cdot}210 \\ 13{\cdot}87 \times {\cdot}0027 &= {\cdot}037 \\ \hline & 37{\cdot}816, \text{ say } 37{\cdot}82 \text{ secs.} \end{aligned}$$

Therefore, for a place in West longitude we must *add* this on to the 12 hrs. 37 min. 29·99 sec., giving 12 hrs. 38 min. 07·81 sec. as the L.S.T. at L.M.N.

(*b*) If the place is in East longitude, we must subtract the 37·82 seconds, giving 12 hrs. 36 min. 52·17 sec. as the L.S.T. at L.M.N. in that case.

Example.—On December 1st, 1914, the G.S.T. at G.M.N. is 16 hrs. 37 min. 59·89 sec. Compute (*a*) the G.S.T. at G.M.N. on December 2nd, (*b*) the L.S.T. at a place in longitude 43° 35′ West at L.M.N. on December 1st.

Ans. (*a*) 16 hrs. 41 min. 56·45 sec.
(*b*) 16 hrs. 38 min. 28·52 sec.

Given the Local Mean Time at any Instant, to Determine the Local Sidereal Time.

The local mean time gives us the interval measured in solar hours, minutes, and seconds, that has elapsed since local noon. We may readily turn this interval into sidereal hours, and so obtain the number of sidereal hours, minutes, and seconds that have elapsed since noon. But in the preceding paragraph we have seen how the L.S.T. at L.M.N. may be determined on any given date at a place in any longitude. Consequently we have only to add to this the number of sidereal hours, minutes, and seconds that have since elapsed, to determine the sidereal time at the instant. We, therefore, proceed as follows :—

1. From the tabulated G.S.T. of G.M.N. on the date in question, compute the L.S.T. of L.M.N. by allowing for difference in longitude.

2. Turn the given L.M.T. into sidereal hours, minutes, and seconds, and add to the L.S.T. of L.M.N.

Example.—*Find the sidereal time at Mount Hamilton (Longitude* 121° 38′ 43·35″ *West) on October 2nd*, 1913, *the L.M.T. being* 9 *hrs.* 17 *min.* 32 *sec. p.m.*

Dividing the longitude by 15, we get the difference in local times between Mount Hamilton and Greenwich to be 8 hrs. 06 min. 34·89 sec.

The gain of the sidereal over the mean time clock in this interval, at the rate of 9·8565 seconds per hour, is 1 min. 19·93 sec.

From the Nautical Almanac, we get G.S.T. at G.M.N. on October 2nd,

1913, ...	12 hrs. 42 min. 23·50 sec.
Add, ...	0 hr. 1 min. 19·93 sec.
L.S.T. at L.M.N., ...	12 hrs. 43 min. 43·43 sec.
But 9 hrs. 17 min. 32 sec. of mean time, when turned into sidereal time, . .	9 hrs. 19 min. 03·59 sec.
Therefore, L.S.T. required, . . .	22 hrs. 02 min. 47·02 sec.

Example.—*Find the sidereal time at Adelaide (longitude* 138° 35′ 04·5″ *E.) on October 2nd*, 1913, *the standard time being* 9 *hrs.* 17 *min.* 32 *sec. p.m.*

The standard time for South Australia is that of the meridian 142½° or 9 hrs. 30 min. E.

Difference in local times between Adelaide and Greenwich = 9 hrs. 14 min. 20·3 sec.

The gain of the sidereal over the mean time clock in this interval at the rate of 9·8565 seconds per hour is 1 min. 31·06 sec.

G.S.T. at G.M.N. on October 2nd, 1913,	12 hrs. 42 min. 23·50 sec.
Subtract, . . .	0 hr. 1 min. 31·06 sec.
L.S.T. at L.M.N., ...	12 hrs. 40 min. 52·44 sec.

The difference between local time and standard time is 15 min. 39·7 sec.

Therefore, the local mean time is . .	9 hrs. 01 min. 52·3 sec.
Turning the interval into sidereal time, we get	9 hrs. 03 min. 21·31 sec.
Therefore, L.S.T. required, . .	21 hrs. 44 min. 13·75 sec.

It is to be particularly noticed that the local mean time must always be reckoned from noon when making such calculations.

Thus, if the mean time is given as 9 hrs. a.m. on October 2nd, this must be reckoned as 21 hrs. October 1st, or 21 hrs. after noon on October 1st.

Given the Sidereal Time at a Place whose Longitude is known, to Determine the corresponding Local Mean Time.

If we can find the sidereal time at mean noon, then by subtracting this from the given sidereal time we find the number of sidereal hours, minutes, and seconds that have elapsed since noon. Turning this interval of time into mean time will give us the number of mean time hours, minutes, and seconds since noon—that is to say, the mean local time required. The rules of procedure are thus :—

1. From the tabulated G.S.T. of G.M.N. on the date in question, compute the L.S.T. of L.M.N. by allowing for difference in longitude.

2. Subtract the L.S.T. of L.M.N. from the given sidereal time. Turn the difference into mean solar time, and the result will be the mean time required.

Example.—*Given that the sidereal time at Mount Hamilton is* 22 *hrs.* 02 *min.* 47·02 *sec. on October* 2*nd*, 1913, *the longitude of the place being* 121° 38′ 43·35″ *West, find the corresponding local mean time.*

As in the first example of the preceding section, we obtain L.S.T. at

L.M.N., ...	12 hrs. 43 min. 43·43 sec.
Given sidereal time, ...	22 hrs. 02 min. 47·02 sec.
Difference, . . .	9 hrs. 19 min. 03·59 sec.

Turning this interval into mean solar time, by the aid of the tables, we get 9 hrs. 17 min. 32 sec. as the L.M.T. required.

EXAMPLE.—*Given that the sidereal time at Adelaide (longitude* 138° 35′ 04·5″ *E.) is* 21 *hrs.* 44 *min.* 13·75 *sec. on October* 2*nd,* 1913, *find the corresponding local mean time.*

As in the second example of the preceding section, we obtain L.S.T.

at L.M.N., ...	12 hrs. 40 min. 52·44 sec.
Given sidereal time, ...	21 hrs. 44 min. 13·75 sec.
Difference, . . .	9 hrs. 03 min. 21·31 sec.

Turning this interval of sidereal time into mean time, we obtain 9 hrs. 01 min. 52·3 sec. as the L.M.T. required.

Alternative Method for Determining the L.S.T., having given the L.M.T.—In the preceding methods for computing L.S.T. from L.M.T. or *vice versa*, it is necessary to first of all compute the L.S.T. of L.M.N., and then to transform another interval of time from mean to sidereal or from sidereal to mean. In the methods about to be described the theory is perhaps a little more complex, but there is only one transformation of a time interval necessary, so that the actual computation is a little shorter.

From the given L.M.T., allowing for the difference of longitude, we readily compute the corresponding mean time at Greenwich. This gives us the interval in mean time that has elapsed since the last Greenwich noon. Turn this interval into sidereal time, and we get the number of sidereal hours, minutes, and seconds that have elapsed since the mean sun was last on the Greenwich meridian.

But from the Nautical Almanac we get the G.S.T. at the last G.M.N. Allowing for the difference in longitude, we can thus obtain the L.S.T. at that instant. And as we have already computed the interval in sidereal time that has since elapsed, we have only to add this on to the L.S.T. at the preceding G.M.N. in order to get the sidereal time required.

We thus get the following rules of procedure :—

1. Allowing for the difference of longitude, compute the mean time at Greenwich at the instant in question, and turn the interval of mean time so found into sidereal time.

2. From the Nautical Almanac obtain the G.S.T. at the previous G.M.N., and allowing for the difference of longitude, determine the corresponding L.S.T. at the same instant.

3. The addition of the results of 1 and 2 gives the L.S.T. required.

As illustrations, for purposes of comparison, we will take the same examples as those already worked.

EXAMPLE.—*Find the sidereal time at Mount Hamilton (longitude* 121° 38′ 43·35″ *West) on October 2nd,* 1913, *the L.M.T. being* 9 *hrs.* 17 *min.* 32 *sec. p.m.*

L.M.T. at Mount Hamilton, . .	9 hrs. 17 min. 32 sec.
Difference due to Longitude (W.), .	8 hrs. 06 min. 34·89 sec.
Corresponding G.M.T., . . .	17 hrs. 24 min. 06·89 sec.

Turned into sidereal time, this is equivalent to 17 hrs. 26 min. 58·41 sec.

From the Nautical Almanac we get G.S.T. at G.M.N. on October 2nd,

1913, ...	12 hrs. 42 min. 23·50 sec.
Difference due to longitude	8 hrs. 06 min. 34·89 sec.
∴ L.S.T. at G.M.N., ...	4 hrs. 35 min. 48·61 sec.
Interval of sidereal time since elapsed .	17 hrs. 26 min. 58·41 sec.
∴ L.S.T. required, ...	22 hrs. 02 min. 47·02 sec.

EXAMPLE.—*Find the sidereal time at Adelaide (longitude* 138° 35′ 04·5″ *E.) on October* 2*nd*, 1913, *the standard time being* 9 *hrs.* 17 *min.* 32 *sec. p.m.*

The standard time for South Australia is that of the meridian 142½° or 9 hrs. 30 min. E.

Standard time at instant, ...	9 hrs. 17 min. 32 sec.
Subtract difference due to longitude, .	9 hrs. 30 min. 0 sec.
Corresponding G.M.T. on *October 1st*, .	23 hrs. 47 min. 32 sec.

Turning the interval into sidereal time we get 23 hrs. 51 min. 26·5 sec.

From the Nautical Almanac we find

G.S.T. at G.M.N. on October 1st, . .	12 hrs. 38 min. 26·95 sec.
Difference due to longitude of Adelaide,	9 hrs. 14 min. 20·3 sec.
∴ L.S.T. at G.M.N. on October 1st, .	21 hrs. 52 min. 47·25 sec.
Interval of sidereal time since elapsed, .	23 hrs. 51 min. 26·5 sec.
∴ L.S.T. required, ...	21 hrs. 44 min. 13·75 sec.

Alternative Method for Determining the L.M.T., having given the L.S.T.—Knowing the longitude of the place, we can compute the sidereal time at Greenwich at the same instant. From the Nautical Almanac, as before, we get the G.S.T. at the previous G.M.N. Subtracting these two results gives us the interval in sidereal time that has elapsed since Greenwich noon.

If we turn this interval into mean solar time, we, therefore, get the interval of mean time that has elapsed since G.M.N. But the L.M.T. corresponding to G.M.N. is readily determined by allowing for the difference of longitude. Adding to this, therefore, the interval of mean time that has since elapsed, we obtain the L.M.T. required.

The principal difficulty arises in places with East longitude, where it may happen that the instant under consideration really precedes noon on the same day at Greenwich. This cannot happen with places having West longitude. If this is the case, it will be at once noticed from the fact that the sidereal time at Greenwich mean noon on the day in question, as found from the Nautical Almanac, will be *less* than the computed Greenwich sidereal time at the instant.

We thus get the following rules for determining the L.M.T., having given the L.S.T. :—

1. Allowing for the difference of longitude, compute the G.S.T. at the instant in question.

2. From the Nautical Almanac find the G.S.T. at the previous G.M.N. and then by subtraction the number of sidereal hours that have elapsed since. Turn this interval of sidereal time into mean time.

3. Add this interval of mean time on to the L.M.T. corresponding to G.M.N., and the result is the L.M.T. required.

EXAMPLE.—*Given that the sidereal time at Mount Hamilton is* 22 *hrs.* 02 *min.* 47·02 *sec. on October 2nd*, 1913, *the longitude of the place being* 121° 38′ 43·35″ *West, find the corresponding L.M.T.*

L.S.T. at Mount Hamilton, . .	22 hrs. 02 min. 47·02 sec.
Difference due to longitude (W.), .	8 hrs. 06 min. 34·89 sec
Corresponding G.S.T., . . .	30 hrs. 09 min. 21·91 sec
G.S.T. at G.M.N., October 2nd, 1913, .	12 hrs. 42 min. 23·50 sec.
Interval of sidereal time since G.M.N., .	17 hrs. 26 min. 58·41 sec.
Equivalent interval of mean time, .	17 hrs. 24 min. 06·89 sec.
L.M.T. corresponding to G.M.N., October 2nd = October 1st, ...	15 hrs. 53 min. 25·11 sec.
∴ L.M.T. required = October 2nd, .	9 hrs. 17 min. 32 sec.

EXAMPLE.—*Given that the sidereal time at Adelaide (longitude* 138° 35′ 04·5″ *E.) is* 21 *hrs.* 44 *min.* 13·75 *sec. on October* 2*nd,* 1913, *find the corresponding L.M.T.*

L.S.T. at Adelaide, ...	21 hrs. 44 min. 13·75 sec.
Difference due to E. longitude, . .	9 hrs. 14 min. 20·30 sec.
Corresponding G.S.T., . . .	12 hrs. 29 min. 53·45 sec.
G.S.T. at G.M.N., October 2nd, 1913, .	12 hrs. 42 min. 23·50 sec.
Instant *precedes* G.M.N. by . .	0 hr. 12 min. 30·05 sec.
Equivalent interval of mean time, .	0 hr. 12 min. 28 sec.
L.M.T. corresponding to G.M.N., October, 2nd, ...	9 hrs. 14 min. 20·30 sec.
∴ L.M.T. required, ...	9 hrs. 01 min. 52·3 sec.

In this case, since the instant precedes G.M.N., we must *subtract* the computed interval of mean time from the L.M.T. corresponding to G.M.N.

Comparison of the Preceding Methods.—As it is a most important thing that the student should thoroughly grasp the principles involved in the transference of time from one system of time measurement to the other, it is a good exercise for him to master both the first method given and the alternative method in each of the preceding cases. The first method, however, involves less thinking and is more mechanical than the other, so that it is the method generally adopted and the one probably most suited for ordinary computations.

Determination of the Local Mean Time of Transit of a Known Star across the Meridian.—One very important application of the preceding work is the calculation of the time of transit of a known star across the meridian, or, as it is commonly termed, the time of culmination.

The Nautical Almanac supplies us with a table of the right ascensions and declinations of the principal stars in the sky, and it has been shown in Chapter II. that the R.A. of a star, expressed in time, is the sidereal time at the moment when the star is on the meridian. Thus the problem is simply that of determining the L.M.T. corresponding to the sidereal time measured by the right ascension of the star. This we may do by one of the methods we have been considering.

EXAMPLE.—*Find the time of culmination of α Triang. Aust. on the evening of August* 17*th*, 1913, *at a place in South Australia whose longitude is* 139° 20′ *E.*, *the time to be measured in the standard time of the meridian* 9 *hrs.* 30 *min. E.*

G.S.T. of G.M.N., August 17th, . .	9 hrs. 41 min. 02 sec.
∴ L.S.T. of L.M.N. at place in longitude 139° 20′ E. computed as in previous work, ...	9 hrs. 39 min. 30·45 sec.
R.A. of α Triang. Aust. = L.S.T. at time of culmination, ...	16 hrs. 39 min. 31 sec.
∴ interval of sidereal time elapsed since L.M.N., ...	7 hrs. 00 min. 00·55 sec.
Equivalent interval of mean time, . .	6 hrs. 58 min. 51·74 sec.
This, therefore, would be the L.M.T. at time of culmination.	
Difference between L.M.T. and time of the standard meridian, . . .	0 hr. 12 min. 40 sec.
∴ Standard time at culmination, . .	7 hrs. 11 min. 31·7 sec.

Time of Transit of the First Point of Aries.—In the preceding work we have adopted the usual practice of effecting the change from sidereal to mean or *vice versa* by means of the column in the Nautical Almanac giving the G.S.T. at G.M.N. But on page 3 of each month there is given another column tabulating for each day in the year the G.M.T. of transit of the First Point of Aries, which may also be used for similar transformation of time. As this instant indicates the beginning of the sidereal day, the column might be appropriately headed, the G.M.T. at sidereal noon.

Given the G.M.T. of Transit of the First Point of Aries, to determine the L.M.T. of Transit at a Place in any other Longitude.

The sidereal clock, as we have seen, is always gaining on the clock keeping mean solar time, at the rate of 9·8565 seconds per mean solar hour, or at the rate of 9·8296 seconds for each sidereal hour. Now the G.M.T. of transit of the First Point of Aries is the reading of the mean time clock when the sidereal clock reads 0 hr. It is the difference between the readings of the two clocks at this instant. As the sidereal clock is gaining on the other this difference will get less as the time increases. Now, at a place in West longitude the transit of the First Point of Aries will take place after an interval of time measured in sidereal hours, minutes, and seconds by dividing the longitude by 15. Thus, when this transit occurs the mean time clock will not be so far ahead of the sidereal clock as it was at Greenwich, and the Greenwich reading of the mean time clock will have to be diminished by subtracting 9·8296 seconds for each hour of longitude.

This reasoning assumes that, whilst different clocks at various places on the earth's surface will have different readings according to the longitude, the difference between the readings of the sidereal and mean time clocks at any place is the same all over the world at the same instant. This must be so according to the reasoning by which we have established the rules for determining the local mean and sidereal times at a place A, having given those at a place B. For we should alter both the sidereal and mean times at B by the same amount, depending on the difference of longitude between B and A, in order to find the corresponding times at A.

Accordingly we get the Nautical Almanac rule for finding from the tables the time of transit of the First Point of Aries at any place. "If the place of observation be not on the meridian of Greenwich, the mean time must be corrected by the *subtraction* of 9·8296 sec. for each hour (and proportional parts for the minutes and seconds) of longitude, if the place be to the West of Greenwich; but by its *addition*, if to the East."

EXAMPLE.—*On August* 1*st*, 1914, *the G.M.T. of transit of the First Point of Aries is* 15 *hrs.* 20 *min.* 28·63 *sec. Compute the local time of transit on the same day* (a) *at a place in longitude* 57° 33′ 28″ *West*, (b) *at a place in the same longitude East.*

(*a*) 57° 33′ 28″ is equivalent in time to 3 hrs. 50 min. 13·87 sec.

$$\begin{array}{rcl} 3 \times 9{\cdot}8296 &=& 29{\cdot}488 \\ 50 \times {\cdot}1638 &=& 8{\cdot}190 \\ 13{\cdot}87 \times {\cdot}0027 &=& {\cdot}037 \\ \hline && 37{\cdot}715, \text{ say } 37{\cdot}72 \text{ seconds.} \end{array}$$

Therefore, for a place in West Longitude, we must subtract this from the 15 hrs. 20 min. 28·63 sec., giving 15 hrs. 19 min. 50·91 sec. as the L.M.T. of transit of the First Point of Aries.

(*b*) For a place in East Longitude we must add the 37·72 seconds, giving 15 hrs. 21 min. 06·35 sec. as the L.M.T. of transit in this case.

EXAMPLE.—Given that the G.M.T. of transit of the First Point of Aries on August 30th is 13 hrs. 26 min. 27·26 min. Find the G.M.T. of transit on August 31st. Find also the local mean time of transit at a place in longitude 45° W.

Ans. 13 hrs. 22 min. 31·35 sec.
and 13 hrs. 25 min. 57·77 sec.

Given the L.S.T. at any Place and the G.M.T. of Transit of the First Point of Aries on the same day, to determine the L.M.T.

The local sidereal time measures the interval in sidereal hours since the transit of the First Point of Aries over that meridian. By turning this, therefore, into mean time hours we get the interval since the transit in mean time hours. But we have just seen how we may calculate the L.M.T. of transit of the First Point of Aries from the information in the Nautical Almanac. The addition of the two results will give us the L.M.T. required. The rule of procedure, therefore, may be expressed:—Turn the given sidereal time into mean time and add it on to the computed L.M.T. of transit of the First Point of Aries.

As the transit of ♈ may take place at any time of the day, some care is necessary in selecting the right transit, as is illustrated in the following example :—

Example.—*Given that the L.S.T. at Mount Hamilton is* 22 *hrs.* 02 *min.* 47·02 *sec. on October* 2*nd*, 1913, *the longitude of the place being* 121° 38′ 43·35″ *West, find the corresponding L.M.T.*

Looking up in the Nautical Almanac the G.M.T. of transit of the First Point of Aries on October 2nd we find it is 11 hrs. 15 min. 45·49 sec. This is very near midnight, and the L.M.T. of transit will not be very different. If we were to add 22 hours on to this it will clearly carry us over into the next day, October 3rd, so that the transit we must select to work from is that on October 1st.

G.M.T. of transit of ♈ on October 1st,	11 hrs. 19 min. 41·39 sec.
Allowance for longitude, to be subtracted,	0 hr. 1 min. 19·71 sec.
L.M.T. of transit of ♈ on October 1st,	11 hrs. 18 min. 21·68 sec.
Mean time equivalent to 22 hrs. 02 min. 47·02 sec. sidereal, . . .	21 hrs. 59 min. 10·32 sec.
∴ L.M.T. required = October 2nd, .	9 hrs. 17 min. 32 sec.

Given the Sidereal Time at Mean Noon at Greenwich to compute the Mean Time at the next Transit of the First Point of Aries.

The Nautical Almanac Columns, one giving the sidereal time at mean noon and the other the mean time of transit of the First Point of Aries, may readily be deduced one from the other.

Thus, suppose the sidereal time at mean noon is denoted by s. Then at noon s sidereal hours have elapsed since ♈ was on the meridian, and, therefore, in $24 - s$ sidereal hours ♈ will again be on the meridian.

If we express $24 - s$ sidereal hours in mean solar time, the result will clearly represent the number of mean solar hours that have then elapsed since noon, and will consequently represent the mean time at the next transit of ♈.

For example, on November 1st, 1913, the sidereal time at Greenwich mean noon is 14 hrs. 40 min. 40·14 sec. Subtracting this from 24 hours, we get 9 hrs. 19 min. 19·86 sec. Turning this into mean solar time, the result is 9 hrs. 17 min. 48·23 sec., which, therefore, represents the mean time at the next transit of ♈.

The converse problem may be dealt with in a similar way.

EXAMPLE.—On October 28th the G.S.T. at G.M.N. is 14 hrs. 23 min. 56·95 sec. Find the mean time of the next transit of ♈.

Ans. 9 hrs. 34 min. 28·67 sec.

Nautical Almanac Data with regard to Time.—In the Nautical Almanac on pages 1, 2, and 3 for each month, various data are given that are useful in time computations. The sidereal time at Greenwich mean noon, the mean time of transit of the First Point of Aries, and the equation of time both for mean and apparent noon, with its rate of variation, are given in each case for every day in the year. In addition, the sun's right ascension is given both for mean and apparent noon. These tabulated results are not all independent, and it is good practice for the student to take a Nautical Almanac and deduce certain of the tabulated values from others that are given. Here are a few of the exercises that may be practised in this way.

1. From the sidereal time at mean noon on one day compute its value for the next day.

2. From the sidereal time at mean noon find the mean time of the next transit of the First Point of Aries.

3. From the mean time of transit of the First Point of Aries determine the sidereal time at mean noon on the same day.

4. From the R.A. of the sun at mean noon, and the

equation of time, with their rates of variation, deduce the sidereal time at mean noon, and the R.A. of the sun at apparent noon.

5. From the sidereal time and the sun's R.A. at mean noon, deduce the equation of time.

Examples.

1. Express in sidereal time the following intervals of mean solar time :—(1) 16 hrs. 15 min. 23 sec., (2) 9 hrs. 17 min. 18·4 sec., and (3) 17 hrs. 52 min. 33·5 sec.

Ans. (1) 16 hrs. 18 min. 3·2 sec.
(2) 9 hrs. 18 min. 49·95 sec.
(3) 17 hrs. 55 min. 29·69 sec.

2. Express in mean solar time the following intervals of sidereal time :—(1) 13 hrs. 22 min. 17 sec., (2) 21 hrs. 35 min. 15·5 sec., and (3) 8 hrs. 55 min. 39·7 sec.

Ans. (1) 13 hrs. 20 min. 05·56 sec.
(2) 21 hrs. 31 min. 43·3 sec.
(3) 8 hrs. 54 min. 11·94 sec.

3. In longitude 148° 15′ E., what is the local mean time corresponding to September 22nd, 4 hrs. 30 min. p.m., standard time of the 150th meridian East of Greenwich ? Find also the corresponding Greenwich mean time.

Ans. (1) 4 hrs. 23 min. p.m.
(2) 6 hrs. 30 min. a.m.

4. Convert Perth apparent time, December 3rd, 4 hrs. 15 min. 20·3 sec. to sidereal time ; also Perth sidereal time, December 3rd, 20 hrs. 26 min. 15·7 sec., to Western Australian standard time (Time of 120th meridian).

Given longitude of Perth, . .	7 hrs. 43 min. 21·7 sec. E.
Sidereal time as G.M.N., Dec. 3rd, .	16 hrs. 47 min. 32·0 sec.
„ „ Dec. 2nd, .	16 hrs. 43 min. 35·5 sec.
Equation of time G.M.N., Dec. 3rd,	10 min. 10·1 sec. to be added to mean time.
„ „ Dec. 2nd,	10 min. 33·5 sec. „

Ans. Sidereal time—
20 hrs. 52 min. 03 sec.
L. Standand time—
3 hrs. 56 min. 02·1 sec.

5. Given that the sidereal time at Greenwich mean noon is 14 hrs. 40 min. 40·14 sec., find the mean time of the next transit of the First Point of Aries.

Ans. 9 hrs. 17 min. 48·23 sec.

6. Given that the mean time of transit of the First Point of Aries at Greenwich is 11 hrs. 19 min. 41·39 sec., compute the sidereal time at Greenwich mean noon on the same day.

Ans. 12 hrs. 38 min. 26·95 sec.

7. The right ascension of a star being 20 hrs. 24 min. 13·72 sec., compute the local mean time of its culmination at Madras (longitude 80° 14′ 19·5″ E.) on September 6th, the sidereal time at Greenwich mean noon on that date being 11 hrs. 2 min. 21·45 sec.

Ans. 9 hrs. 21 min. 12·8 sec.

8. Convert 22 hrs. 22 min. 44·58 sec. sidereal time at Greenwich, January 20th, 1913, into mean time, given that the mean time of transit of the First Point of Aries on January 19th is 4 hrs. 6 min. 14·36 sec.

Ans. 2 hrs. 25 min. 18·96 sec.

9. Find the mean local time corresponding to 5 hrs. 17 min. 32 sec. sidereal time at Moscow (longitude 37° 34′ 15″ E.), given that the sidereal time of Greenwich mean noon on the same day was 23 hrs. 54 min. 52 sec.

Ans. 5 hrs. 22 min. 11 sec

10. Find the standard time of culmination of α Centauri at Adelaide on June 1st, 1914, R.A. = 14 hrs. 33 min. 49 sec., longitude = 9 hrs. 14 min. 20·3 sec. Standard meridian 9 hrs. 30 min. E. G.S.T. at G.M.N. on the same date = 4 hrs. 36 min. 30·1 sec.

Ans. 10 hrs. 12 min. 51·3 sec.

11. Find the local mean time of the transit of δ Crucis over the meridian, at a place in longitude 11 hrs. 30 min. E. on the 10th May, 1913. Transit First Point of Aries, G.M.T., 9th May, 20 hrs. 49 min. 48·44 sec.; star's R.A., 12 hrs. 10 min. 33·08 sec.

Ans. 9 hrs. 00 min. 14·9 sec.

12. The mean time of transit of the First Point of Aries for January 21st, 1911, is given in the Nautical Almanac as 4 hrs. 00 min. 24·79 sec. For the same date the R.A. of α Leonis is given as 10 hrs. 03 min. 38·76 sec. Find the exact local mean time when α Leonis passed the meridian of a place in longitude 135° E.

Ans. 2 hrs. 03 min. 53·13 sec. a.m.,
January 22nd.

13. Compute the local sidereal time at noon by standard time at Adelaide on October 24th, 1914, given

Longitude of Adelaide, . . . 9 hrs. 14 min. 20·30 sec. E.
Longitude of standard meridian, . 9 hrs. 30 min. E.
G.S.T. at G.M.N., October 23rd, . 14 hrs. 04 min. 14·18 sec.

Ans. 13 hrs. 50 min. 57·40 sec.

14. In the forenoon of August 1st, 1914, at Melbourne, longitude 9 hrs. 39 min. 54 sec. E., a mean time chronometer was compared with a sidereal clock known to be 14·6 seconds fast on true local sidereal time. It was found—

Time by sidereal clock, . . . 8 hrs. 18 min. 09·00 sec.
Time by chronometer, . . . 11 hrs. 41 min. 34·32 sec.

The data in the appended table is taken from the Nautical Almanac :—

GREENWICH MEAN NOON.

Date—1914.	Apparent R.A. of Sun.	Variation in One Hour.	Equation of Time to be subtracted from Mean Time.	Variation in One Hour.
	Hrs. Mins. Secs.	Secs.	Mins. Secs.	Secs.
July 31, .	8 39 18·14	9·749	6 14·54	0·108
Aug. 1, .	8 43 11·80	9·723	6 11·65	0·134
Aug. 2, .	8 47 04·84	9·697	6 08·13	0·159

Determine

(*a*) The sidereal time at Greenwich mean noon, August 1st.
(*b*) The R.A. of the sun at apparent noon, August 1st.
(*c*) The error of the mean time chronometer on Victorian Statute time (meridian 10 hrs. E.).

Ans. (*a*) 8 hrs. 37 min. 0·16 sec.
(*b*) 8 hrs. 43 min. 12·80 sec.
(*c*) 0 hrs. 21 min. 04·05 sec. slow.

CHAPTER VI.

THE LOCATION OF OBJECTS ON THE CELESTIAL SPHERE.

In order that the surveyor may pick out and observe a particular star with a theodolite, it is frequently necessary, more especially when he wishes to make the observation in daylight or evening twilight, that he should know the altitude and azimuth of the star at the given time. From the Nautical Almanac he obtains its right ascension and declination, and from these data he has to compute altitude and azimuth. In this chapter we will deal with this problem and show how, given the position of a star in one system of co-ordinates we may determine its co-ordinates in another.

A. **Knowing the Latitude and Time at the Place of Observation and the Right Ascension and Declination of a particular Star, it is required to determine its Altitude and Azimuth.**

In Fig. 16, let P be the pole, S the star, Z the zenith, A Z P B the plane of the meridian.

Draw the great circle through Z and S to intersect the horizon in H.

If we know the local mean time we can compute the corresponding sidereal time by the methods of the last chapter. But we have seen that the right ascension of the star is the same thing as the sidereal time at the moment of the star's transit across the meridian. Consequently the difference between the sidereal time at the instant of observation and the right ascension of the star gives the interval in sidereal time between the moments of the star's transit across the meridian and of observation—that is to say, it gives, when turned into degrees, minutes and seconds, the hour angle of the star S P Z. If the sidereal time at the moment of observation is less than the right ascension of the star, the difference measures the angle S P Z towards the East of the meridian, if the right ascension is the less, the angle is measured toward the West.

Thus, in the spherical triangle Z S P, we know Z P, the complement of the latitude, and S P, the polar distance of the star which is the complement of the declination, and the included angle Z P S.

From these data we can compute the third side Z S,

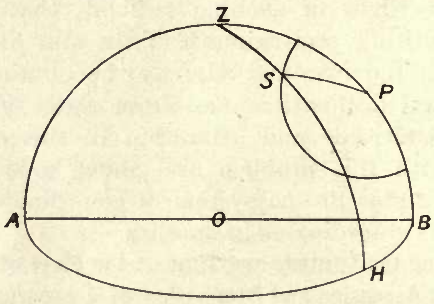


Fig. 16.

which is the zenith distance of the star, or the complement of the altitude, and the angle S Z P, which determines the azimuth.

Calling the angles of the spherical triangle Z, P, and S respectively, the formulæ applicable to the solution of a spherical triangle, having given two sides and the included angle, are

$$\tan \tfrac{1}{2}(S + Z) = \frac{\cos \tfrac{1}{2}(ZP - SP)}{\cos \tfrac{1}{2}(ZP + SP)} \cdot \cot \tfrac{1}{2} P,$$

$$\tan \tfrac{1}{2}(S - Z) = \frac{\sin \tfrac{1}{2}(ZP - SP)}{\sin \tfrac{1}{2}(ZP + SP)} \cdot \cot \tfrac{1}{2} P.$$

From these equations we compute the angles S and Z. Then, to determine S Z, we have

$$\sin S\,Z = \frac{\sin P \sin S\,P}{\sin Z}.$$

EXAMPLE.—*At a place in South Australia in longitude* 9 *hrs.* 14 *min. E., latitude* 32° 35′ *S., it is required to determine the altitude and azimuth of Achernar at* 7 *p.m. standard time on December* 1*st,* 1913. *The R.A. of Achernar is* 1 *hr.* 34 *min.* 33 *sec., and its declination South is* 57° 40′ 33″.

The standard time of South Australia is that of the meridian 9 hrs. 30 min. E.

The Greenwich time corresponding to 7 p.m. standard time on December 1st is thus 21 hrs. 30 min. on November 30th.

Therefore, the interval of time which has elapsed since Greenwich noon on November 30th is 21 hrs. 30 min. of mean time, equivalent to 21 hrs. 33 min. 31·9 sec. of sidereal time.

From the Nautical Almanac, the sidereal time at Greenwich noon on

November 30th is	16 hrs. 35 min. 0·3 sec.
Difference due to longitude, . . .	9 hrs. 14 min. 0 sec.
Local sidereal time at Greenwich noon, .	1 hr. 49 min. 0·3 sec.
Interval of sidereal time since elapsed, . .	21 hrs. 33 min. 31·9 sec.
Local sidereal time required, ...	23 hrs. 22 min. 32·2 sec.

This gives us the sidereal time at the instant of observation.

But the R.A. of Achernar is 1 hr. 34 min. 33 sec.

Thus Achernar lies 21 hrs. 47 min. 59·2 sec. to the West of the meridian, or 2 hrs. 12 min. 0·8 sec. to the East.

Multiplying this by 15, we get the hour angle of the star as 33° 0′ 12″ to the East.

Referring now to Fig. 16, we have

$$Z\,P = \text{co-latitude} = 57^\circ\ 25'$$
$$P\,S = \text{complement of declination} = 32^\circ\ 19'\ 27''$$
$$P = 33^\circ\ 0'\ 12''$$

$\cos \frac{1}{2}(Z\,P - S\,P) = \cos 12^\circ\ 32'\ 46{\cdot}5''$, . .	9·9895036
$\cot \frac{1}{2} P = \cot 16^\circ\ 30'\ 6''$, . .	10·5283488
	10·5178524
$\cos \frac{1}{2}(Z\,P + S\,P) = \cos 44^\circ\ 52'\ 13{\cdot}5''$, . .	9·8504650
$\tan \frac{1}{2}(S + Z)$, ...	10·6673874

$\therefore \frac{1}{2}(S + Z) = 77° 51' 40''$.

$\sin \frac{1}{2}(ZP - SP)$	$= \sin 12° 32' 46{\cdot}5''$, . .	9·3369150
$\cot \frac{1}{2} P$	$= \cot 16° 30' 6''$, . . .	10·5283488
		9·8652638
$\sin \frac{1}{2}(ZP + SP)$	$= \sin 44° 52' 13{\cdot}5''$, . .	9·8485005
$\tan \frac{1}{2}(S - Z)$,	. . . .	10·0167633

$\therefore \frac{1}{2}(S - Z) = 46° 6' 20''$

$\therefore Z = 31° 45' 20''$.

Thus the star lies in the direction 31° 45′ 20″ East of South.

To find its altitude,

$\sin P$	$= \sin 33° 0' 12''$, ...	9·7361477
$\sin SP$	$= \sin 32° 19' 27''$, ...	9·7281173
		9·4642650
$\sin Z$	$= \sin 31° 45' 20''$	9·7212303
$\sin SZ$,	. . . .	9·7430347

$\therefore SZ = 33° 36' 1''$.

Therefore, the altitude of the star is the complement of this, or 56° 23′ 59″.

Very commonly for such calculations it is sufficient to compute the position of the star to the nearest minute, and in that case five-figure logarithms are sufficient.

B. **Having observed the Altitude and Azimuth of a Star, the Time of Observation being noted, it is required to determine its Right Ascension and Declination.**

The latitude and longitude of the place of observation are supposed known.

Then in the figure, Z being the zenith point, P the pole, and S the star, as before.

In the spherical triangle Z S P, Z P is known, being the co-latitude ; Z S, the zenith distance, is also known, and the angle S Z P, which the vertical plane passing through the star makes with the meridian.

Thus we know two sides and the included angle, and the triangle may be solved to find S P and the angle S P Z.

The formulæ to be used are those of the preceding problem.

$$\tan \tfrac{1}{2}(S+P) = \frac{\cos \frac{1}{2}(ZP - ZS)}{\cos \frac{1}{2}(ZP + ZS)} \cot \tfrac{1}{2} Z,$$

$$\tan \tfrac{1}{2}(S-P) = \frac{\sin \frac{1}{2}(ZP - ZS)}{\sin \frac{1}{2}(ZP + ZS)} \cot \tfrac{1}{2} Z,$$

$$\sin SP = \frac{\sin Z \,.\, \sin ZS}{\sin P}.$$

The angle S P Z, being turned into hours, minutes, and seconds, at the rate of 15° for one hour, measures the sidereal time that will elapse before S comes to the meridian if S is to the East, or the interval of sidereal time since S was on the meridian if it is to the West.

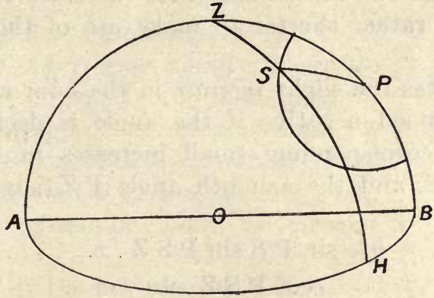


Fig. 17.

But the right ascension of the star is the sidereal time when it is on the meridian.

Therefore, to obtain the right ascension of the star, add the time value of the angle S P Z to the local sidereal time at the moment of observation if the star is to the East of the meridian, and subtract it if the star is to the West.

The declination of the star is, of course, the complement of the computed polar distance S P.

C. **Having computed the Altitude and Azimuth of a Star for a Given Time of Observation, it is required to determine its Approximate Position at some Short Interval of Time afterwards.**

When a surveyor is preparing for daylight observations of a star, it will be generally necessary for him to take at least two readings of its position. To give him time to read the verniers and reverse the instrument before taking the second observation, he requires to know the altitude and azimuth of the star at an interval of five or ten minutes after the first reading.

The computation for the second position may, of course, be made in precisely the same way as we have already done for the first, in which case several of the logarithms already taken out will be useful for the calculation.

But it is rather shorter to make use of the following formulæ :—

If x denotes the slight *increase* in the hour angle S P Z (to be reckoned negative if the angle is decreasing), y and z the corresponding small increases in the zenith distance Z S, and the azimuth angle P Z S respectively. Then

$$y = \sin \mathrm{P\,S} \sin \mathrm{P\,S\,Z} \,.\, x. \qquad . \qquad . \quad (1)$$

$$z = -\frac{\cot \mathrm{P\,S\,Z} \,.\, y}{\sin \mathrm{Z\,S}}. \qquad . \qquad . \qquad . \quad (2)$$

The values of P S Z and Z S to be used in the equations being those found in the first calculation.

To establish the formulæ, let A B C (Fig. 18) be a spherical triangle. Then if b and c remain unchanged, we require to find the small changes y and z in a and B respectively if the angle A is increased by a small amount x.

By the ordinary formulæ for spherical triangles we have

$$\cos a = \cos b \cos c + \sin b \sin c \cos \mathrm{A}$$

$$\text{and } \cos (a + y) = \cos b \cos c + \sin b \sin c \cos (\mathrm{A} + x)$$

Subtracting gives

$$\cos a \cos y - \sin a \sin y - \cos a = \sin b \sin c$$
$$(\cos \mathrm{A} \cos x - \sin \mathrm{A} \sin x - \cos \mathrm{A}).$$

Now, if x and y are very small, we can, if they are measured in circular measure, replace $\sin x$ and $\sin y$ by x and y respectively, and put $\cos x$, $\cos y$ each equal to unity. Doing this, we get

$$-y \sin a = -\sin b \sin c \sin A \,.\, x.$$

Putting $\sin c \sin \mathrm{A} = \sin \mathrm{C} \sin a$, this becomes

$$y = \sin b \sin \mathrm{C} \,.\, x,$$

which is the first formula given.

Since we have here simply the ratio of y to x, the result will hold good in whatever system of measurement y and x are expressed, provided they are both measured in the same system, both in degrees or both in circular measure.

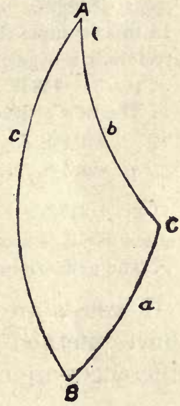


Fig. 18.

Further, by the law of sines,

$$\frac{\sin (\mathrm{B} + z)}{\sin b} = \frac{\sin (\mathrm{A} + x)}{\sin (a + y)}.$$

Expanding and substituting as before, we get

$$(\sin \mathrm{B} + z \cos \mathrm{B})\,(\sin a + y \cos a) = \sin b\,(\sin \mathrm{A} + x \cos \mathrm{A})$$

and $$\sin \mathrm{B} \,.\, \sin a = \sin b \,.\, \sin \mathrm{A}.$$

$\therefore$ substracting, and neglecting the product of two small quantities y and z,

$$z \sin a \cos \mathrm{B} + y \cos a \sin \mathrm{B} = x \sin b \cos \mathrm{A}.$$

Putting $$x = \frac{y}{\sin b \sin C}$$

$$z \sin a \cos B = y \left(\frac{\cos A}{\sin C} - \sin B \cos a\right) = -y \frac{\cos B \cos C}{\sin C}.$$

$$\therefore z = -y \,.\, \frac{\cot C}{\sin a}, \text{ which is the second formula.}$$

To illustrate the application of the formulæ we will extend the scope of the example already worked out in Section A of this Chapter, and compute the position of Achernar 5 sidereal minutes after 7 p.m.

From the previous work the angle P S Z = 123° 58′, P S = 32° 19′, Z S = 33° 36′.

sin P S, ...	9·72803
sin P S Z, ...	9·91874
·44338, ...	$\bar{1}$·64677

In this example the hour angle of the star is measured to the East, and, therefore, x is negative, and = − 5 minutes of time = − 1° 15′ of arc.

∴ y = − ·44338 × 75′ = − 33′.

∴ The new altitude is 56° 24′ + 33′ = 56° 57′.

cot P S Z.	9·82844
sin Z S, ...	9·74303
1·2173, ...	0·08541

and cot P S Z is negative, ∴ z = 1·2173 × (− 33′) = − 40′.

∴ The new azimuth is 31° 45′ − 40′ = 31° 5′ East of South.

If results are only required to the nearest minute, the above method is quite sufficient, provided the small differences are not much more than 2 degrees of arc.

Examples.

1. Compute to the nearest minute of arc the altitude and azimuth of Sirius (dec. = 16° 35′ South, R.A. = 6 hrs. 41 min.) at a place in latitude 31° 57′ South at 12 hrs. sidereal time.

Ans. Azimuth = 260° 51′.
Altitude = 17° 12′.

2. Compute the altitude and azimuth of Sirius 10 sidereal minutes later than in 1.

Ans. Azimuth = 259° 38′
Altitude = 15° 7′.

3. At a place in latitude 28° South at 1 hr. 37 min. sidereal time, the altitude of Canopus is observed as 33° 3′ and its azimuth as 136° 44′. Compute the R.A. and dec. of the star.

Ans. R.A. = 6 hrs. 21 min. 58 sec.
Dec. = 52° 38′ 48″ S.

4. What is the angular distance between the stars A (R.A., 4 hrs. 23 min. 53 sec., Dec., 16° 04′ 25″ N.) and B (R.A., 2 hrs. 54 min. 34 sec., dec., 40° 08′ 03″ N.)?

Ans. 30° 54′ 14″.

5. Find the angular distance between A (R.A., 19 hrs. 42 min. 11 sec., dec., 8° 23′ 52″ N.) and B (R.A., 22 hrs. 47 min. 41 sec., dec., 30° 33′ 17″ N.).

Ans. 59° 06′ 04″.

6. If the N. dec. of a star is 40°, show that the number of hours in the sidereal day during which it will be below the horizon of a place which has latitude 30° N. is 8·136.

CHAPTER VII.

ASTRONOMICAL AND INSTRUMENTAL CORRECTIONS TO OBSERVATIONS OF ALTITUDE AND AZIMUTH.

Parallax.—The fixed stars are so distant from us that their directions always appear to be the same, no matter from what point upon the earth's surface they are observed. Even with our most refined instruments no difference can be detected, because their distance is practically infinitely great in comparison with the diameter of the earth. But with the members of our own system, the sun, the moon, and the planets, we are dealing with bodies incomparably nearer to us, and their relative positions amongst the fixed stars of the sky are not precisely the same when viewed from different places. It is, therefore, essential that their registered right ascensions and declinations should be referred to some definite point upon the earth, in order that they may be available to all observers. The point selected is the earth's centre, because, having observed the direction of a planet from any station on the earth's surface, it is an easy matter to deduce its position as it would appear at the earth's centre, and conversely if the position of the star is tabulated as it would be seen from the centre of the earth we may readily find its position as seen from any place on the earth's surface. The selection of the earth's centre as the imaginary place of observation greatly simplifies the computations, and consequently most astronomical observations of bodies in our own solar system are reduced so as to show what the result would be if the observation could have been at the centre of the earth. The registered right ascensions and declinations of the Nautical Almanac are those the different bodies would have if viewed from the earth's centre.

The difference between the directions of a heavenly body as seen from the earth's centre and as seen from the place of observation is known as its *Parallax.*

Thus, as in Fig. 19, if S is the sun or planet observed,

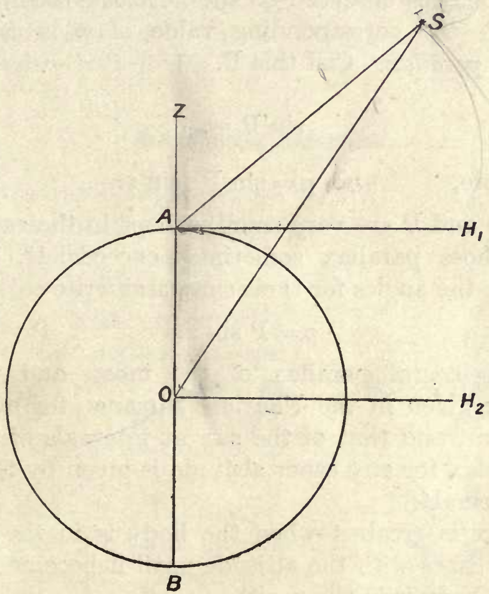


Fig. 19.

A the point of observation, and O the earth's centre, the parallax of the body is the angle A S O, the difference in the directions of A S and O S. If A H_1 is the direction of the horizontal at A, the altitude of S is the angle S A H_1. If O H_2 is drawn parallel to A H_1, then the difference of the angles S O H_2 and S A H_1 = the difference of the angles S O B and S A B which = the angle A S O. Thus, if we call p the parallax, $p =$ angle A S O = S O H_2 − S A H_1. Clearly the angle S O H_2 is always greater than the angle S A H_1.

If z denotes the zenith distance of S as observed from A, r the earth's radius O A, and d the distance O S, then,

From the triangle A O S, $\frac{\sin p}{\sin z} = \frac{r}{d}$.

If the body is observed on the horizon—that is to say, if $z = 90°$—the corresponding value of p is called the *horizontal parallax*. Call this P.

Then $$\sin P = \frac{r}{d}.$$

Therefore, $$\sin p = \sin P \cdot \sin z.$$

Since p and P are very small, except in the case of the moon, whose parallax sometimes exceeds 1°, we may substitute the angles for their sines and write

$$p = P \sin z.$$

The horizontal parallax of the moon and principal planets is given in the Nautical Almanac for every day in the year, and that of the sun at intervals of 10 days. The parallax for any other altitude is given by the above simple formula.

Parallax is greatest when the body is in the horizon, and diminishes with the altitude until it becomes nothing when the body is in the zenith.

We see from Fig. 19 that the effect of the parallax upon a celestial object is to make its altitude appear less when observed from A than it would be if seen from O. Consequently, when reducing observations to the earth's centre, we must *add* the correction for parallax observed to the altitude, or

True altitude = observed altitude + parallax.

Parallax has no effect upon the azimuth of an object in the sky; the correction is made to altitude only.

This statement is strictly correct only when the earth is regarded as a perfect sphere. If the spheroidal form of the earth is taken into account there will be parallax in azimuth as well as in altitude. Even then, however, the correction in azimuth is too small to be worth considering except in the case of certain special lunar observations.

The horizontal parallax of the sun ranges between 8·65 and 8·95 seconds. At an altitude of 60° its parallax is reduced to half of this.

Atmospheric Refraction.

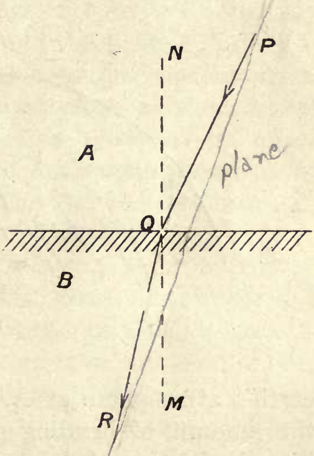


Fig. 20.

When a ray of light passes from one medium into a denser medium as from air into water or from air into glass, it is bent or *refracted* towards the normal to the bounding surface. Thus, as in Fig. 20, if a ray of light passes from the medium A to a denser medium B, traversing the path P Q R, the refracted ray Q R will always make a smaller angle with the normal to the separating surface than the incident ray P Q. The direction of bending is always such that the bent or refracted ray lies in the same plane as that passing through the incident ray P Q and the normal Q N. The law governing the amount of bending is that the ratio between the sines of the angles P Q N and R Q M is constant for these particular media and the value of this ratio is known as the *coefficient of refraction.*

Similarly, when a ray of light from a celestial body reaches the atmosphere surrounding the earth, it is bent slightly out of its original path. If the atmosphere were a uniform homogeneous medium with a definite upper surface it would be comparatively easy to determine the precise amount of bending of the ray. But the density of the atmospheric air diminishes with the height above the earth's surface. Consequently a ray from a star S (Fig. 21), when it reaches the upper limit of the

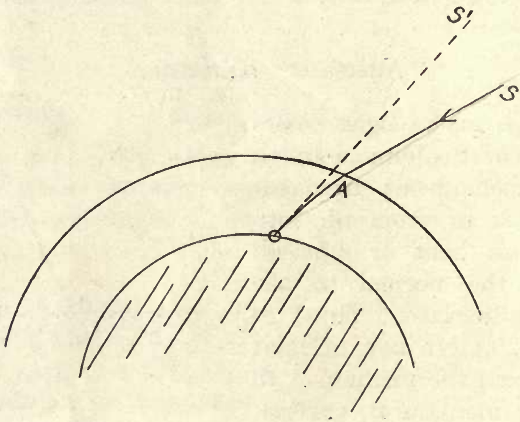


Fig. 21.

earth's atmosphere at A, is only very slightly bent, but the amount of bending gradually increases as it passes into the lower and denser layers of air. Its path from A to an observer on the earth's surface at O is thus a curve, and the ray ultimately reaches the observer, so that it appears to him to come in the direction of O S^1. Thus, the observer sees the star apparently at S^1 in the celestial sphere, whereas in reality the star is at S. The effect is that the star is apparently raised above its true position, and its apparent altitude is greater than the true altitude if it could be observed from O with no intervening atmosphere. The observed altitude of a celestial body must, therefore, be corrected in order to deduce its true altitude, the correction being always *subtracted* from the observed altitude. The amount of bending of the ray varies somewhat with the pressure and temperature of the air, but it is greatest for stars on the horizon, and gradually decreases to nothing for a star in the zenith. For a body on the horizon the mean value of the correction is 33′—that is to say, a star will be just visible on the horizon when it is really 33′ below it. Thus the sun, whose diameter is about 32′, is visible just above the horizon when it is in reality just below it.

It will be seen from the figure, since the refracted ray always lies in the plane containing the incident ray S A, and the normal to the spherical bounding surface at A, that S and S′ will lie in the same plane as the vertical at O. This means that refraction produces its effect entirely in altitude, and has no influence upon the apparent azimuth of a heavenly body. Thus no correction in azimuth is necessary on account of refraction.

As we do not know the exact laws which govern the pressure and temperature of the earth's atmosphere at different heights, nor even the distance to which it extends around the earth, no satisfactory computation of the amount of refraction at different altitudes can be made from theoretical considerations alone. By making different assumptions as to the character of the earth's atmosphere various formulæ have been derived, but as their demonstration generally requires mathematics of a rather advanced character, we shall not attempt the problem here. In any case, as we cannot be sure of the correctness of the assumptions that have to be made in order to derive the formula, the values of the constants used have to be obtained and checked from actual observations. There are various ways by which the amount of refraction at different altitudes may be actually measured, and for practical purposes that formula is selected which best fits the results of such measurements.

The formula that has found most favour, and which has been most used by astronomers for this purpose, is that of Bessel,

$$r = A\,(B\,t)^{M}\,T^{N}\cot a,$$

where $a =$ the apparent altitude,

$r =$ the amount of refraction in seconds of arc,

B, a factor depending on the height of the barometer,

t, a factor depending on the reading of the thermometer attached to the barometer,

T, a factor depending on the reading of a thermometer so exposed as to give the temperature of the external air.

A, M, and N are factors depending on the altitude of the celestial body.

When suitable values are given to the different factors, this formula can be made to fit in with the results of actual observations on refraction with great precision, and where great accuracy is required this is the formula that is most generally adopted. To use the formula it is, of course, put into the logarithmic form—

$$\log r = \log A + M\,(\log B + \log t) + N \log T + \log \cot a,$$

and the values of M, N, log A, log B, log t, and log T are obtained from appropriate tables. Such a table is published in Chambers' Mathematical Tables.

The constants M and N in the above formula do not differ sensibly from unity if the altitude is considerable. If these are taken each $= 1$, the formula may be put into a form which makes the application of tables much simpler. For the values of B, t, and T are each unity for certain particular values of the barometric height, and for certain special temperatures of the attached and unattached thermometers. Consequently for this particular condition of the atmosphere, which we may take as the standard condition, we have $r = \text{A} \cot a$.

If now we denote by r_1 the amount of the refraction for any other temperature and pressure, we have—

$$r_1 = \text{A} \,.\, \text{B}\, t \,.\, \text{T} \cot a,$$
$$\therefore\ r_1 = \text{B} \times t \times \text{T} \times r,$$

or refraction = the refraction for altitude a under the standard or mean conditions multiplied by the factors B, t, and T, depending on the height of the barometer and the temperatures recorded by the attached and unattached thermometers.

A table of refractions constructed for standard conditions of the atmosphere is commonly termed a table of *mean* refraction. With the aid of such a table and subsidiary tables for B, t, and T, we may first of all find the value of the "mean refraction" for the measured altitude, then pick out the values of B, t, and T for the particular conditions of the atmosphere, and the true refraction = the mean refraction $\times$ B $\times\, t \times$ T.

This is the method of determining the refraction most commonly adopted for ordinary purposes, and gives accurate enough results unless the altitude is very small. The necessary tables are in Chambers' Mathematical Tables.

For many purposes, and more especially for high altitudes, it is quite sufficiently accurate to use the value of the refraction as given in the mean refraction table. The refraction is always less than $1'$ if the altitude is greater than $45°$, and for zenith distances up to $20°$ the refraction is practically $1''$ per $1°$.

Corrections to Observations on Account of Residual Instrumental Errors.

It forms no part of the purpose of this book to enter upon a discussion of the construction of the ordinary instruments of the surveyor and the methods of adjustment. These are matters dealt with in text-books on Surveying. It will be assumed that the reader is acquainted with the construction of the surveyor's transit theodolite and with the usual methods of securing its accurate adjustment. But even when the adjustments have been made with great care, there commonly remain certain residual errors which affect the accuracy of the celestial observations, and must be taken into account if the best results are to be obtained. Of these, the two most important are, (1) an error due to the fact that the line of collimation of the telescope is not accurately at right angles to the transverse axis about which the telescope turns, and (2) an error produced if this transverse axis is not absolutely horizontal. We will consider the effect of each of these in turn.

The Effect of an Error of Collimination.—Let us suppose that the line of collimation of the telescope, instead of being accurately at right angles to the axis about which the telescope turns, is in error by a small angle c; that is to say, the telescope makes an angle $90° - c$ on one side and $90° + c$ on the other side with the axis. On turning the telescope about the transverse axis, which is adjusted so as to be horizontal, the line of collimation would, if in accurate adjustment, trace out a vertical plane passing through the zenith. But if in error, and the line of collimation is not at right angles to the axis, then, as it is plunged up and down, it will trace out a conical surface and on the celestial sphere it will trace out a circle parallel to a vertical circle through the zenith. Thus, as in Fig. 22, if there were no collimation error the line of collimation of the telescope would trace out the great circle Z S′ N, but if in error it will sweep out the parallel small circle L S M. Now, suppose that the star S is observed in such a telescope, and let S S′ be an arc of a great circle drawn at right angles to Z N. S S′ = N M = c the collimation error.

If we draw the great circle arc Z S, then Z S is the true zenith distance of the star. But the observed zenith distance is Z S′. Similarly the correct azimuth is measured by the angle H Z S, whereas the azimuth as read on the instrument is H Z S′.

In the right-angled triangle S S′ Z, S S′ being denoted by c, we have

$$\cos S Z = \cos S' Z \cos c.$$

If c is very small, as should be the case if the instrument is in decent adjustment, we may take $\cos c = 1$, and, therefore, practically $S' Z = S Z$, or no correction will

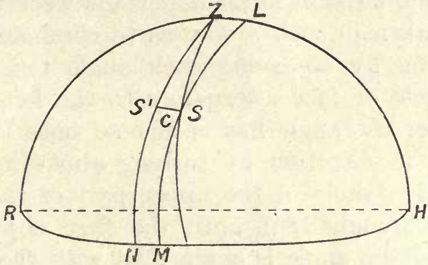


Fig. 22.

usually be necessary to the observed zenith distance or altitude.

Also, denoting by Z the angle S Z S′, the error in azimuth, we have

$$\sin c = \sin S Z \,.\, \sin Z,$$

and since c and Z are both small, we may write

$$Z = c \,.\, \operatorname{cosec} S Z,$$

or the error in azimuth = the collimation error multiplied by the cosecant of the zenith distance.

The error in azimuth thus becomes very great if the star is near the zenith, but is $= c$ for a star on the horizon.

The following table shows the way in which the error varies with the altitude of the star :—

Error in Azimuth corresponding to a Collimation Error c for Various Altitudes of Object.

Altitude of Star,	0°	30°	60°	70°	80°	85°	89°
Error in azimuth,	c	$1{\cdot}15c$	$2c$	$2{\cdot}92c$	$5{\cdot}76c$	$11{\cdot}47c$	$57{\cdot}3c$

The Elimination of Instrumental Errors by Changing Face.—Although we have in the preceding paragraph investigated the effect of a given collimation error, it is very seldom that the surveyor will need to take this error into account, because in all important work the observations are taken in such a way as to eliminate its effects. This is done by observing each angle twice, with the vertical circle or face alternately to the left and to the right. After the angle has been read once the telescope is reversed in direction by turning about its horizontal axis, and the whole of the upper part of the theodolite is turned through 180° until the first object is again sighted, and the angle is again read with the instrument in this reversed position. The operation is commonly referred to as "changing face," and should be adopted in all theodolite observations, as it gives a means both effectual and simple of eliminating the chief instrumental errors. An error in collimation will not affect the horizontal angle between two objects if both are at the same altitude, but if the altitudes are different, then if the collimation error makes the measured angle a little too great when the vertical circle is facing the left it will make it just as much too small when the vertical circle faces the right, and thus the mean of the two readings gives the correct result.

Now, when measuring the azimuth of a star, we have to sight the telescope to a *moving* object, and it is not possible, therefore, to exactly repeat the measurement because in the interval of time taken in changing face the position of the star is slightly changed. But it is characteristic of all the more accurate methods of astronomical measurement suitable for the surveyor, that reliance is never placed upon one observation, but the methods are so arranged that a series of observations can be made at short intervals, the face of the instrument being alternately changed from right to left, so that a mean may be obtained from which instrumental errors are largely eliminated.

The Error made if the Transverse Axis of the Telescope is not truly Horizontal.—This error, just as that due to collimation with which we have just dealt, may also

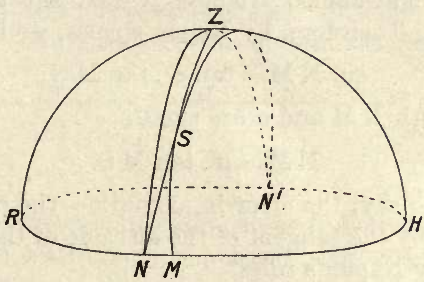


Fig. 23.

be largely eliminated by the method of changing face. But in this case the elimination is not so perfect, and as it is an easy matter by means of a striding level to actually measure the departure of the axis from the horizontal at each observation, it is frequently desirable to observe the error and allow for it in the computation.

If the axis of the telescope is not truly horizontal, the line of collimation, when the telescope is turned about the axis, will not trace out a great circle in the sky passing through the zenith, as it should do, but will trace out a great circle inclined to the vertical. Thus in Fig. 23, if N Z N^1 denotes the great circle that would be traced out in the celestial sphere if the axis were horizontal, N S N[1] denotes the circle actually traced out if the axis is inclined at a small angle a. Let S be a star observed with this telescope, and draw the great circle Z S M passing through the zenith and the star.

The angle Z N S $= a$.

The actual observed altitude of the star is measured by the arc N S, whereas the true altitude is given by the arc M S.

Again, the azimuth of the star is actually measured on the circle of the horizon from the point N, whereas it should be measured from the point M. So that the error in azimuth is the angular measure of the arc M N.

In the right-angled triangle N S M, the angle S N M $= 90° - a$. Therefore, by Napier's rules, we have

$$\sin \mathrm{N\,M} = \tan a \,.\, \tan \mathrm{M\,S},$$

or, since both N M and a are small,

$$\mathrm{N\,M} = a \,.\, \tan \mathrm{M\,S}.$$

That is to say, the error in azimuth = the error in level multiplied by the tangent of the altitude of the star.

Again, by Napier's rules,

$$\sin \mathrm{M\,S} = \sin \mathrm{N\,S} \,.\, \cos a,$$

and since a is small and $\cos a$ may be taken = 1, it follows that we may take M S = N S, which means that no appreciable correction has to be made to altitude. The error produced is practically in azimuth only.

The error in azimuth increases with the altitude of the star. It is zero on the horizon, becomes $= a$ for an altitude of 45°, and is very great for stars near the zenith.

Error in Azimuth corresponding to a Level Error a in the Axis for Various Altitudes of Object.

Altitude of star,	0°	30°	45°	60°	70°	80°	85°	89°
Error in azimuth,	0	$0{\cdot}58a$	a	$1{\cdot}73a$	$2{\cdot}75a$	$5{\cdot}67a$	$11{\cdot}43a$	$57{\cdot}3a$

Determination of the Level Error of the Axis by means of the Striding Level.—In order to make practical application of the correction just investigated, it is necessary to actually measure the level error of the transverse axis of the theodolite for each observation. This is readily done by means of the striding level, a very sensitive spirit level supported by two legs with V bearings at the bottom, which can rest upon each end of the transverse axis of the theodolite. The tube of the level is marked off in divisions, the values of which are known or may be readily determined by test. The graduations read outwards from the centre towards both ends. To eliminate errors of construction the readings should be taken in pairs, the striding level being read first in one position and then reversed on its bearings with each observation. Both ends of the bubble are read on each

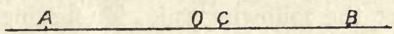


Fig. 24.

occasion, the observer standing so as to face the direction in which the instrument is pointed. He reads first the left-hand end, then the right, then reverses the level and reads again.

Suppose, as in Fig. 24, the bubble extends from A to B, O being the centre of the graduations and C the middle point of the bubble.

Then C B = half the length of the bubble

$$= \frac{OA + OB}{2}.$$

$$\therefore OC = OB - CB = OB - \frac{OA + OB}{2}$$

$$= \frac{OB - OA}{2}.$$

This, therefore, measures the deflection of the centre of the bubble from its normal position, and, when multiplied by the value of 1 division of the level, gives the angular measure of the deflection from the horizontal.

Suppose that the readings of the left-hand and right-hand ends of the bubble are l_1 and r_1 respectively before reversal and l_2 and r_2 after reversal of the level on its bearings. Then, according to the first reading, the error of the axis is $\frac{l_1 - r_1}{2}$, and according to the second reading $\frac{l_2 - r_2}{2}$. Thus the mean determination is

$$\frac{l_1 + l_2 - (r_1 + r_2)}{4}.$$

We thus get the following rule, for finding the error in level of the horizontal axis, after a series of striding level readings taken in this way. Add up the left-hand readings. Add up the right-hand readings. Subtract the two sums and divide by the total number of readings. The result is to be multiplied by the value of the level graduation in seconds of arc.

If the striding level were perfect in construction, then the reading obtained on reversal should be the same as that given previously. $\frac{l_1 - r_1}{2}$ should $= \frac{l_2 - r_2}{2}$. Any difference is due to an error in the striding level, and is equal to twice the striding level error. Thus the error of the striding level itself $= \frac{l_1 - r_1 - (l_2 - r_2)}{4}$.

For example, if the left-hand readings of the bubble of the striding level are 6·3 and 4·8, the corresponding right-hand readings being 5·2 and 6·8, we proceed as follows :—

L.	R.
6·3	5·2
4·8	6·8
11·1	12·0
	11·1
	4) 0·9
	0·22

Therefore, if one division on the level corresponds to 14″ inclination, the angle the axis makes with the horizontal is $0{\cdot}22 \times 14 = 3{\cdot}1''$.

In this case the sum of the readings to the right is greater than the sum of the readings to the left, and, therefore, the right-hand end of the axis is the higher. This would mean that the azimuth of a star (measured from the North towards the right) would appear to be greater than it really is, and the correction to be made would consequently have to be subtracted. If the left-hand end of the axis were the higher the correction to azimuth would have to be added.

If the preceding readings were taken with the striding level on the transverse axis of a theodolite when a star was being observed at an elevation of 42° 33′ and the azimuth reading was 127° 33′ 10″, the correction to be made to azimuth would be $3{\cdot}1 \times \tan 42° \, 33' = 3{\cdot}1 \times {\cdot}918 = 2{\cdot}8''$, and the corrected azimuth would be 127° 33′ 7·2″.

Allowance for Error of Alidade Level.—In most modern theodolites intended for astronomical observations, no level is attached to the telescope itself, but instead a delicate level, known as the alidade level, is attached to the vernier or microscope arms of the vertical circle, and the circle turns with the telescope so that when the telescope is horizontal the verniers are at zero.

With this form of instrument, when reading vertical angles, each reading should be repeated by changing the face of the instrument, and to allow for any slight departure from true horizontality in the setting of the theodolite, the alidade level should be read on each occasion. In this case the readings of the two ends of the bubble are commonly referred to as O and E, according as they are at the object or eye end of the telescope.

The principle involved is exactly the same as that of the striding level just described. The error in level will be found by dividing the difference between the sums of the readings of the object end and eye end by the total number of readings, and then multiplying the result by the angular value of one division of the scale of the spirit level. If the readings of the object end are greater than those of the eye end, then the zero line is pointing slightly upwards, and the correction must be added on to the observed altitude. If the readings of the eye end are the greater, then the correction is to be subtracted. So that

$$\text{Correction to altitude} = + \frac{O - E}{\text{number of readings}} \times \text{value of 1 division.}$$

Thus, suppose that two observations are taken, one with the face of the instrument to the left and the other with face right, as follows :—

	O.	E.
F. L.	5	9
F. R.	7	7
	12	16
		12
		4) 4
		1

Thus, if the angular value of one division on the level is 14″, it will follow that the altitude measured must be reduced by this amount.

Clearly this correction applies to vertical angles only, and does not affect the measurement of horizontal angles.

CHAPTER VIII.

THE DETERMINATION OF TRUE MERIDIAN.

The determination by observation of a true North and South line is a very important and common operation for the surveyor, and there are many ways in which it may be done. In practice, however, preference is given to such methods as will allow a set of observations to be taken so that instrumental errors may be eliminated, some readings being taken with F.R. (face right) and others with F.L. (face left), and also to such methods as do not require too great an interval of time between the observations. There is an objection to methods which require stars to be sighted at an interval of several hours, not only on the score of practical convenience, but because the atmospheric refraction may have changed considerably in the time that has elapsed. We shall confine our attention to the principal methods in actual use.

Referring Mark.—When determining the azimuth of a star or other celestial object, it is necessary to have a referring mark whose azimuth may be measured with respect to that of the star, so that the true direction may be found of a fixed reference object. It is commonly indicated in field notes by the letters R.M. It is highly desirable that there should be no need to refocus the telescope after pointing it to a heavenly body and then directing it to the referring mark, and this requires that the referring mark should be where practicable about a mile away. When stellar observations are being taken the referring mark should be made to imitate the light of a star as nearly as possible. This may be done with a bull's eye lantern placed in a box or behind a screen, through which a small circular hole is cut to admit the light to the observer. The face of the screen may be painted with stripes, so that it may be readily observable in the day time. If the referring mark is not to appear larger than a star in the field of view of the telescope, the diameter of the hole must not be more than about a third of an inch at a distance of one mile. Some observers prefer a narrow vertical slit in the screen, and others use a larger hole with two cross wires at right angles to each other.

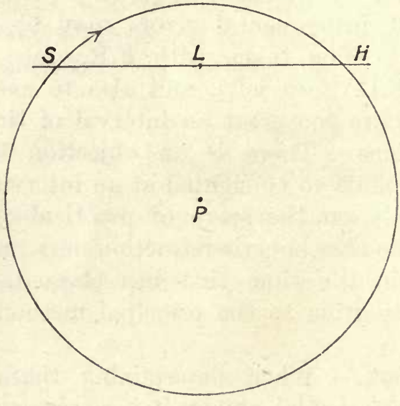


Fig. 25.

First Method—By Equal Altitudes of a Circumpolar Star.—To mark out a true North and South line, we have to determine the direction of the celestial pole, and the simplest method is probably that of observing a circumpolar star at equal altitudes. No calculations are necessary, and no knowledge of the latitude, longitude, or local time is required by the observer.

If the circle in Fig. 25 represents the circular path of a star round the pole, the problem is to determine the direction of the centre P of this circle. Suppose that the star is observed at S, and then, keeping the angle of elevation of the telescope unchanged, the observer waits until he sees the star again at H at the same altitude. Clearly the point L, midway between S and H, will be vertically above the pole P, and all that the observer has to do to get his true meridian is to bisect the angle between S and H. Nothing could be simpler in principle, but certain precautions are necessary to get accurate results.

In the first place, when fixing either the points S or H, we are really marking the point of intersection of the horizontal line with the circle. Now, we can fix the intersecting point of two lines most accurately when the two lines are at right angles, and so the best position for the line S H is when it passes somewhere near P. As the star takes 24 sidereal hours to complete its circle round the pole, this would mean that the second observation would be made about 12 hours after the first. This would be often impossible and generally inconvenient. If, on the other hand, the line S H is taken too near the top of the circle, the star is moving so rapidly in a horizontal direction that it is not possible to secure good intersections.

Two simple observations at S and H, such as we have just described, would not be sufficient to enable instrumental errors to be eliminated, and so in practice a set of at least four observations are made, as illustrated in Fig. 26. They will be made somewhat as follows :—Set the instrument to zero and point to the R.M. Point to the star in the position S_1, measuring the horizontal angle between S_1 and the R.M., and noting also the altitude of S_1. Then change the face of the instrument and point again to the star, which will by this time be at S_2. Again note horizontal angle and altitude. Keeping the telescope clamped at the same vertical angle, unclamp the upper plate and move the telescope round, waiting until the star is again seen in the position S_3. When the star is got into the field of view of the telescope, the upper plate is again clamped and the star followed by means of the tangent screw until it again coincides with the centre of the cross wires. Having read the horizontal angle, the face of the instrument is again changed, the altitude of the telescope is again set to the reading at S_1, and the star is again followed until at S_4 it once more is

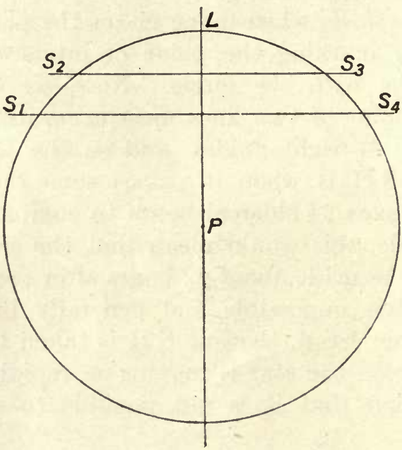


Fig. 26.

in the centre of the field. Finally, the telescope is pointed to the R.M.

The direction midway between S_2 and S_3 should, of course, if there are no errors, coincide with that midway between S_1 and S_4. This will not usually be the case, but the mean of the two results is taken and instrumental errors are largely eliminated.

If a, b, c, and d be the angles which S_1, S_2, S_3, and S_4 make with the R.M., then if the R.M. be outside the angle subtended by S_1 S_4 at the observer's eye, the angle that the R.M. makes with the true meridian will be

$$\frac{a+b+c+d}{4}.$$

If, on the other hand, the direction of the R.M. lies between S_1 and S_4, the angle will be

$$\frac{a+b-(c+d)}{4}.$$

The reason of this difference will be seen from Fig. 27, where O P represents the true meridian bisecting the

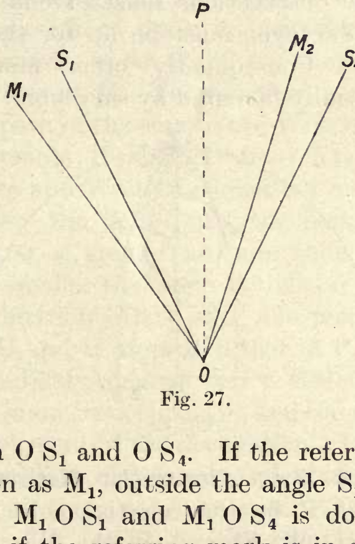


Fig. 27.

angle between O S_1 and O S_4. If the referring mark is in such a position as M_1, outside the angle S_1 O S_4, the *sum* of the angles M_1 O S_1 and M_1 O S_4 is double the angle M_1 O P. But if the referring mark is in such a position as M_2, within the angle S_1 O S_4, the *difference* of the angles M_2 O S_1 and M_2 O S_4 is double the angle M_2 O P.

The polar distances of the stars are not absolutely constant, as the theory of the method assumes, but undergo very slight changes during the year, which are tabulated in the Nautical Almanac. In the course of 24 hours, however, the alteration never amounts to more than a small fraction of a second of arc, and, therefore, need not be considered.

An unknown error may be introduced by changes in the atmospheric refraction during the considerable interval of time that must separate the first and second sets of observations. The method will give results quite sufficiently accurate, however, for the ordinary purposes of the surveyor, it may be carried out without the use of mathematical tables or Nautical Almanac, and it involves no knowledge of the position of the observer. Its great practical disadvantage is the length of time over which the observations must extend, and to carry them out the surveyor must be up for the greater part of the night. Consequently other more convenient methods are usually favoured by surveyors.

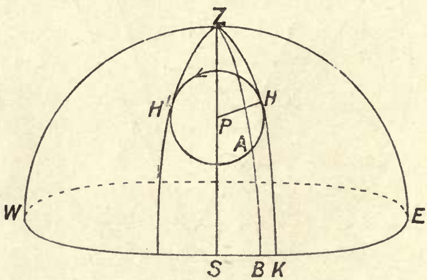


Fig. 28.

Second Method—By a Circumpolar Star at Elongation.—In Fig. 28, let P be the celestial pole, Z the zenith of the observer, W, S, and E the West, South, and East points respectively on the circle of the horizon, or the West, North, and East points according as the observer is in the Southern or Northern Hemisphere. The small circle with P as centre represents the path of a circumpolar star A. The vertical plane passing through the zenith of the observer and the star traces out the circle Z A B on the celestial sphere. This will be the circle swept out by the telescope of a theodolite when the telescope, after being directed to the star, is turned in a vertical plane about its transverse axis. As the star moves from the position shown in the figure this vertical plane will make a greater and greater angle with the plane of the meridian Z P S until the star arrives at the position H, where the vertical circle Z H K, swept out by the telescope, is a tangent to the circular path of the star. This is the point where the vertical plane containing the star makes its greatest angle with the plane of the meridian. At this point the star is said to be *at elongation*, and, clearly, its motion being then vertical, it is in a favourable position for observations upon its azimuth, because its horizontal movement is so slight for some time before and after it arrives at H. There will be a corresponding point H′ in the path of the star to the West of the celestial pole, and the points H and H′ are referred to as the points of Eastern and Western elongation respectively.

It is clear from the figure that the points H and H′ will always be at a greater altitude than the celestial pole P, but the smaller the circle of the star's path or the greater the declination of the star, the more nearly will the altitude of H and H′ approach that of P.

Now, if a Nautical Almanac star is selected for observation, we shall know its declination, and the polar distance P H is the complement of the declination. If, in addition, we know the latitude of the place of observation, then, in the right-angled spherical triangle Z P H, we shall know P H and Z P, which is the complement of the latitude. Hence, by Napier's rules, we can compute the angle P Z H. We have

$$\text{Sin } PH = \sin ZP \sin PZH$$

or

$$\text{Sin } PZH = \cos \text{declination} \times \text{sec. latitude}.$$

This calculation gives us the angle that the star at H makes with the meridian. Hence, if we measure the angles that the star at H makes with some referring mark, the azimuth of the R.M. is determined.

The method so far indicated would require the direction of the star to be measured at the exact moment of elongation. But we have set it down as a general principle that at least two observations should be made, one with F.L. and the other with F.R., and it bcomes important to enquire what error in azimuth will be made if sufficient time is taken to obtain two readings.

On making the necessary calculations, it will be found that, for a place in latitude 30°, the azimuths of stars at different polar distances will not alter by 5″ after the moment of elongation until the following times have elapsed :—

Polar Distance of Star.	Time after Moment of Elongation before Azimuth changes by 5″.
10°,	3 min. 33 sec.
15°,	3 min. 7 sec.
20°,	2 min. 35 sec.
30°,	2 min. 11 sec.

As there will be a corresponding and nearly equal period before elongation, it follows that for a star whose polar distance is 10° there will be a total time of about 7 minutes during which its motion is so nearly vertical that the total change of azimuth in that period is not more than 5″. For a star whose polar distance is 30°, the corresponding period is $4\frac{1}{3}$ minutes.

If, then, the surveyor, as will commonly be the case in ordinary work, is not seeking to determine the true meridian nearer than within 20″, it will be quite sufficiently accurate to take two observations of the star, one with F.L. and the other with F.R., not at the exact moment of elongation, but one just before and the other probably just after elongation. The time required to read both verniers, reverse face, and set the telescope again on the star should not be more than three or four minutes, so that there should be time to get both observations within the period we have just calculated during which the azimuth of the star does not alter by 5″. The nearer the star is to the pole the greater the length of time available for the observations.

The average value of the angle that the star makes with the meridian, as determined by two observations in this way, is clearly always a little less than the angle at elongation. In order to get the most accurate results with this method, it is better not to use the formula for the star at elongation at all, but to get a careful set of four observations of the star *near* elongation, observing the altitude of the star at each measurement. In Fig. 29, let A represent the star moving in its circular path round

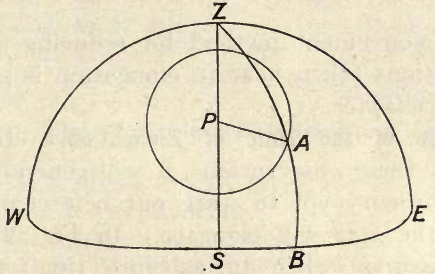


Fig. 29.

the pole P, Z the zenith, Z A B the vertical circle passing through the zenith and the star. Then, in the triangle Z P A, if the altitude of the star is measured, the values of Z A (90° – the altitude) and Z P (the co-latitude) and P A (the polar distance of the star) are known. If

$$
\begin{aligned}
P\,A &= p = \text{polar distance of star},\\
P\,Z &= c = \text{co-latitude},\\
Z\,A &= z = \text{zenith distance},\\
s &= \tfrac{1}{2}(p + c + z),
\end{aligned}
$$

$$\sin \tfrac{1}{2}\, P\,Z\,A = \sqrt{\frac{\sin(s-z)\sin(s-c)}{\sin z \sin c}},$$

$$\text{or } \log \sin \tfrac{1}{2}\, P\,Z\,A = \tfrac{1}{2}\{\log \sin(s-z) + \log \sin(s-c) + \log \operatorname{cosec} z + \log \operatorname{cosec} c\}.$$

Such a set of observations should be made in the following order :—Point to R.M., point to star, reading altitude and horizontal angle, reverse face, and point to star again. Turn back to R.M. and read angle. Then another pair of observations are made in the same way. The mean of the first two observations and the mean of the second two are then used as the data for two separate computations of the azimuth of the R.M. by means of the formula we have just given. The average of the two results, if the work is carefully done, will give a very accurate determination. This is the method recommended in the Hand Book of Instruction for Western Australian Surveyors. An example is given a little further on.

A more convenient method for reducing any number of observations taken near to elongation is given at the end of this chapter.

Calculation of the Time of Elongation.—In order to prepare for these observations, it will generally be necessary for the surveyor to work out beforehand the time at which the star will elongate. In Fig. 28 the angle Z P H measures, when turned into time, the sidereal time that must elapse before the star at H comes on to the meridian. But when the star is on the meridian the sidereal time is given by the R.A. of the star. Thus the sidereal time when the star is at H is = the R.A. of the star − the hour angle of the star Z P H. This sidereal time has then to be turned into mean time by the methods we have previously discussed.

EXAMPLE.—*To find the time of Eastern elongation of* β *Centauri on April* 10*th*, 1914, *at a place in S. lat.* 31°, *longitude* 135° *E.*

R.A. of β Centauri, . . .	13 hrs. 57 min. 47·7 sec.
Dec. of β Centauri, . . .	59° 57′ 43·8″ S.

We first of all find the time of culmination, the local sidereal time at that instant being given by the R.A. of the star. Thus at culmination—

Local sidereal time, . . .	13 hrs. 57 min. 47·7 sec.
Corresponding Greenwich sidereal time, ...	4 hrs. 57 min. 47·7 sec.
Sidereal time at G.M.N., April 10th,	1 hr. 11 min. 29·19 sec.
Interval in sidereal time after Greenwich noon, ...	3 hrs. 46 min. 18·5 sec.
Interval in mean time after Greenwich noon, ...	3 hrs. 45 min. 41·4 sec.
Local time corresponding to G.M.N., April 10th, ...	9 hrs. 0 min. 0 sec.
∴ Local mean time at culmination,	12 hrs. 45 min. 41·4 sec.

We have now to find the time from elongation to culmination, which will be measured (Fig. 28) by the angle Z P H. From the right-angled triangle Z P H in that figure we have

$$\cos ZPH = \tan PH \, . \cot ZP = \cot \text{dec.} \times \tan \text{lat.}$$

cot dec. = cot 59° 57′ 43·8″, . .	9·7621015
tan lat. = tan 31°, ...	9·7787737
cos 69° 40′ 10″, ...	9·5408752

∴ angle Z P H = 4 hrs. 38 min. 40·66 sec. sidereal time = 4 hrs. 37 min. 55 sec. mean time.

∴ time of Eastern elongation = 12 hrs. 45 min. 41·4 sec. — 4 hrs. 37 min. 55 sec. = 8 hrs. 7 min. 46·4 sec., April 10th.

Time of Western elongation = 12 hrs. 45 min. 41·4 sec. + 4 hrs. 37 min. 55 sec. = 17 hrs. 23 min. 36·4 sec., April 10th, or 5 hrs. 23 min. 36·4 sec. a.m. on April 11th.

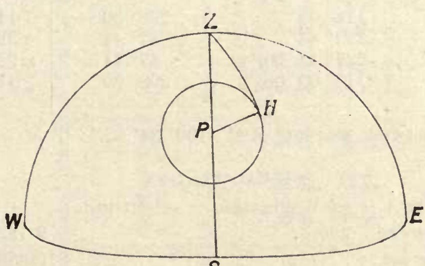


Fig. 30.

Azimuth, Altitude, and Hour-Angle at Elongation. — In Fig. 30, if P denotes the celestial pole, Z the zenith of the observer, and H a star at elongation. In the right-angled triangle Z P H we have Z P = the co-latitude, P H = the star's polar distance or the complement of the declination, the angle Z P H = the hour angle of the star, P Z H = the azimuth of the star if P is the North celestial pole or the supplement of the azimuth if P is the South celestial pole, Z H = the star's zenith distance or the complement of the altitude. Hence we have the following relations :—

$$\cos ZPH = \cot \text{dec.} \times \tan \text{lat.}$$
$$\sin PZH = \cos \text{dec.} \times \sec \text{lat.}$$
$$\sin \text{altitude} = \text{cosec dec.} \times \sin \text{lat.}$$

Example of Observation of Star at Elongation for Azimuth.

Star—Canopus. *Date*—June 26th, 1914.
R.A.—6° 22′ 01·07″. *Place*—Survey Office Tower, Adelaide.
Declination—52° 38′ 48″. *Latitude*—34° 55′ 38″.
R.M. taken—*Obelisk* on Mt. Lofty.

Computed approximate Standard Time of W. elongation, 4 hrs. 14 min.

Observations.

Object.	Face.	Horizontal Circle.		
		A.	B.	Mean.
R.M.	R	118° 09′	298° 09′	118° 09′ 00″
Star	R	227° 44′	47° 44′	227° 44′ 00″
Star	L	227° 43′ 30″	47° 44′	227° 43′ 45″
R.M.	L	118° 09′ 00″	298° 09′	118° 09′ 00″

Mean angle between star and R.M., 109° 34′ 52″. R.M. to East of Star.

Calculation.

Formula.—Sin A = cos declination × sec latitude.

log cos dec., ...	9·7829945
log sec lat., ...	10·0862497
log sin A, ...	9·8692442
A from South, ...	47° 44′
Azimuth of Star, ...	227° 44′
Angle between R.M. and Star, . .	109° 34′ 52″
Bearing of R.M., ...	118° 09′ 08″

EXAMPLE OF OBSERVATION OF STAR NEAR ELONGATION FOR AZIMUTH.

Date, . . .	27th June, 1914.	Barometer, . . .	29·5 inches.
Star, . . .	Canopus.	Thermometer, . .	62°.
R.A., . . .	6 hrs. 22 min. 01 sec.	Place, ...	Adelaide.
Declination, . .	52° 38′ 47″.	Latitude, . . .	34° 55′ 38″ S.
		Longitude, . . .	9 hrs. 14 min. 20 sec. E.

Computed Standard Time of Eastern Elongation, 8 hrs. 29 min. 38·75 sec. a.m.

Object.	Face.	Standard Time of Observation.			Horizontal Circle.			Vertical Circle.		
		H.	M.	S.	A.	B.	Mean.	A.	B.	Mean.
R.M.	R				118° 09′ 00″	298° 08′ 50″	118° 08′ 55″			
R.M.	L				298° 09′ 40″	118° 09′ 30″	118° 09′ 35″			
Star	L	8	26	35	312° 17′ 00″	132° 16′ 40″	132° 16′ 50″	45° 42′ 30″	45° 42′ 20″	45° 42′ 25″
Star	R	8	28	28	132° 16′ 40″	312° 16′ 30″	132° 16′ 35″	45° 58′ 40″	45° 59′ 00″	45° 58′ 50″
Star	R	8	29	33	132° 16′ 00″	312° 16′ 00″	132° 16′ 00″	46° 08′ 30″	46° 08′ 20″	46° 08′ 25″
Star	L	8	31	29	312° 17′ 30″	132° 17′ 00″	132° 17′ 15″	46° 24′ 00″	46° 24′ 20″	46° 24′ 10″
R.M.	L				298° 09′ 30″	118° 09′ 20″	118° 09′ 25″			
R.M.	R				118° 09′ 20″	298° 09′ 20″	118° 09′ 20″			

Mean angle between R.M. and star—

From first pair, ... 14° 07′ 27·5″

From second pair, ... 14° 07′ 15″

COMPUTATION FOR AZIMUTH.

	First Pair.	Second Pair.
Observed altitude,	45° 50′ 37·5″	46° 16′ 17·5″
Refraction,	55″	54″
Corrected altitude,	45° 49′ 42·5″	46° 15′ 23·5″
Zenith distance z,	44° 10′ 17·5″	43° 44′ 36·5″
Co-latitude c,	55° 04′ 22″	55° 04′ 22″
Polar distance p,	37° 21′ 13″	37° 21′ 13″
$2s$,	136° 35′ 52·5″	136° 10′ 11·5″
s,	68° 17′ 56″	68° 05′ 06″
$s - c$,	13° 13′ 34″	13° 00′ 44″
$s - z$,	24° 07′ 38·5″	24° 20′ 29·5″
L sin $(s - z)$,	9·6114752	9·6150814
L sin $(s - c)$,	9·3594456	9·3524891
L cosec z,	10·1568864	10·1602513
L cosec c,	10·0862497	10·0862497
L sin² $\frac{1}{2}$ Z,	19·2140569	19·2140715
L sin $\frac{1}{2}$ Z,	9·6070284	9·6070357
$\frac{1}{2}$ Z,	23° 51′ 58″	23° 52′ 00″
Z,	47° 43′ 56″	47° 44′ 00″
Azimuth of star,	132° 16′ 04″	132° 16′ 00″
Angle to R.M.,	14° 07′ 27·5″	14° 07′ 15″
Azimuth of R.M.,	118° 08′ 36·5″	118° 08′ 45″

Mean azimuth of R.M., ... 118° 08′ 41″.

CALCULATION OF TIME OF ELONGATION.

G.S.T. of G.M.N., June 27th, 1914,	6 hrs. 19 min. 0·63 secs.
Allowance for longitude,	1 min. 31·06 secs.
L.S.T. of L.M.N.,	6 hrs. 17 min. 29·57 sec.
R.A. of Canopus or L.S.T. of Culmination,	6 hrs. 22 min. 01·07 sec.
Sidereal interval since L.M.N.,	4 min. 31·50 sec.
Converted to mean solar time,	4 min. 30·75 sec.
Correction to standard time,	15 min. 40 sec.
Standard time of Culmination,	12 hrs. 20 min. 10·75 sec. p.m.

cos hour angle at elongation = cot dec. × tan lat.

	L cot dec. (52° 38′ 47·25″)	= 9·8826803
	L tan lat. (34° 55′ 38″)	= 9·8440521
		9·7267324
∴ hour angle	= 57° 47′ 28″	
equivalent to . .	3 hrs. 51 min. 9·87 sec. sidereal interval	
or	3 hrs. 50 min. 32 sec. mean time interval	
subtract from . .	12 hrs. 20 min. 10·75 sec.	
giving	8 hrs. 29 min. 38·75 sec. a.m. as the standard time of the Eastern elongation.	

The Effect of an Error in the Latitude.—In the preceding calculations we require to know the declination of the star and the latitude of the place. The declination of the star is given by the Nautical Almanac, but it is possible that the latitude may not be known with the same degree of precision. In Fig. 30, we have

$$\sin Z \cos l = \cos d, \quad . \quad . \quad . \quad (1)$$

where $l =$ latitude, $d =$ declination, $Z =$ angle P Z H.

Suppose that a small change y in the latitude produces an alteration x in the azimuth Z, d remaining unaltered.

Then $$\sin (Z + x) \cos (l + y) = \cos d.$$

Expand each of these terms, remembering that x and y are small, so that $\sin x$, $\sin y$ may be replaced by x and y respectively, and $\cos x$, $\cos y$ by unity. We then get

$$(\sin Z + x \cos Z)(\cos l - y \sin l) = \cos d.$$

Subtracting (1) from this equation and neglecting the term involving the product of x and y

$$x \cos Z \cos l - y \sin Z \sin l = o,$$

or $$x = y \tan l \tan Z$$

$$= y \tan l \frac{\sin Z}{\sqrt{(1 - \sin^2 Z)}}$$

$$= y \tan l \frac{\cos d}{\sqrt{(\cos^2 l - \cos^2 d)}} \quad \text{from (1)}$$

Thus $x = o$ if $l = o$, and $x = \propto$ if $l = d$. l cannot be greater than d, because if so Z P is less than P H (Fig. 30), and the formulæ would not apply. For such a star there is no position of greatest elongation, as the azimuth of the star during its revolution completes the circle of the compass. If $l = d$, the path of the star passes through the zenith.

The following table gives the values of the error in azimuth compared to the error in latitude, as calculated by the preceding formula, for various values of l and d.

RATIO OF ERROR IN AZIMUTH TO SMALL ERROR IN LATITUDE.

Declination of Star Observed.	In Latitude 20°.	In Latitude 30°.	In Latitude 40°.
60°	·22	·4	·7
70°	·14	·24	·4
80°	·06	·1	·19

In the cases tabulated an error in latitude of, say, 5″ will produce an error in azimuth of less than 5″, the tabulated ratios being all less than 1. The error in azimuth may, however, be much greater than the error in latitude, if the star observed has a declination approaching the value of the latitude.

In any given latitude, the error is least when the star selected is nearest to the pole. From the formula, $x = o$ if $d = 90°$. This and other considerations, as we have seen, all point to the desirableness of selecting a star for observation as near to the celestial pole as possible.

Star Observations in Daylight.—It is often a very great convenience to the surveyor to be able to make his observations for meridian in the day time. The method that we have just described of taking observations on a star at or near elongation may be used perfectly well in daylight, provided that a sufficiently bright star is selected. Such work is done most easily in the late afternoon.

The following are suitable stars for such daylight observations in the Southern Hemisphere :—

α Argus (Canopus), α Eridani (Achernar),
α^2 Centauri, β Centauri, α^1 Crucis.

As these stars cannot be seen with the naked eye in daylight, it is necessary to compute the position of the one selected for observation before directing the telescope to it. The time of elongation may be computed by the method already discussed, and the azimuth and altitude of the star at elongation determined by the formulæ given. When these calculations are made the star may be readily picked up.

To select the most suitable star, compute roughly the sidereal time when it is desired to make the observations. A star must be selected which culminates some 4 or 5 hours before or after this. That is to say, the star chosen must have a right ascension some 4 or 5 hours greater or less than the computed sidereal time.

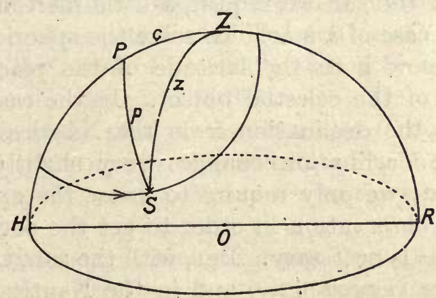


Fig. 31.

Third Method—Extra-Meridian Observations on Sun or Star.— Suppose, in Fig. 31, that S denotes any heavenly body which moves in a circle round the celestial pole P. Let Z be the zenith of the observer. Then if the altitude of S is observed at any instant, and if in addition we know the latitude of the place and the declination of the celestial body, then in the spherical triangle P Z S we know the three sides $SZ = z = 90° -$ altitude, $PZ = c = 90° -$ latitude, $PS = p = 90° -$ declination. Consequently, we can determine the angle Z which the vertical plane through S makes with the true meridian.

If we write $s = \frac{1}{2}(p + c + z)$, then

$$\sin \tfrac{1}{2} Z = \sqrt{\frac{\sin (s - z) \cdot \sin (s - c)}{\sin z \cdot \sin c}},$$

or

$$\cos \tfrac{1}{2} Z = \sqrt{\frac{\sin s \cdot \sin (s - p)}{\sin z \cdot \sin c}},$$

from which

$$\log \sin \tfrac{1}{2} Z = \tfrac{1}{2} \left\{ \log \sin (s - z) + \log \sin (s - c) + \log \operatorname{cosec} z + \log \operatorname{cosec} c \right\},$$

and similarly for the second formula.

A more detailed discussion of these formulæ is given in the account of extra-meridian observations for time.

The method may be applied either to a star or to the sun, but for the sun we require a little more information than in the case of a star. To solve the spherical triangle we must know both the latitude of the place and the declination of the celestial object. In the case of a star we can get the declination from the Nautical Almanac, and as the declination changes very slightly throughout the year, we only require to know the approximate date of the observation in order to get the declination as accurately as is necessary. But, with the sun, the declination changes very rapidly, and in the Nautical Almanac its value is given at Greenwich mean noon for every day in the year. In order to obtain the declination at any other instant, we must know the Greenwich time at the moment in question, and this means that we must know both the local mean time and the longitude. An error of one minute in the time may produce an error of 1″ in the sun's declination. With the sun, therefore, the time of observation must be noted as well as the altitude.

The method is also well suited for daylight observations upon stars, as the very brightest stars are available for this class of observation. Sirius (magnitude $-1{\cdot}4$) is a very suitable star.

If the observation is made in the Northern Hemisphere and S is to the East of the meridian, the angle P Z S is the azimuth of the celestial body. If S is to the West of the meridian, the azimuth $= 360° -$ P Z S.

If the observer is in the Southern Hemisphere, then the azimuth $= 180° -$P Z S or $180° +$ P Z S, according as S is to the East or to the West of the meridian.

Extra Meridian Observations of a Star.—At least two measurements of the altitude and the horizontal angle made with the R.M. should be taken, one with the F.L. and the other with F.R. Since the mean refraction for objects at an altitude of 45° is 57″, it is necessary to correct for refraction in the measurement of the altitude. As the proper correction for refraction is somewhat uncertain for stars anywhere near the horizon, the star selected for observation should have an altitude of at least 30°. The order of procedure should be as follows:—

Point the telescope to the R.M.

Turn the upper part of the instrument round so as to direct the telescope to the star, reading both verniers on the horizontal circle. Measure also the altitude of the star.

Reverse the face of the instrument.

Again point telescope to star, measuring horizontal angle and altitude.

Turn the upper part of the instrument, this time in the reverse direction, until the telescope points to the R.M.

In the interval between the two pointings to the star it will have moved considerably in altitude. If we average the two altitudes and with the value so obtained solve for Z by the formula given, the result will give us the azimuth corresponding to this mean altitude, but that is not exactly the same thing as the mean of the azimuths in the two observed positions. Provided, however, that the difference in altitude of the star at the two observations is not more than one or two degrees, the error thus made is so slight that it is not worth considering.

When the observations are to be made upon a bright star in the day time, it will be necessary, first of all, to compute the azimuth and altitude of the star for the time of the first observation in the manner explained and illustrated in Chapter VI. The azimuth and altitude 5 or 10 minutes later may then be deduced, as shown in the same chapter.

Extra Meridian Observations upon the Sun. — The sun, being an object of large size in the field of view of the telescope, cannot be observed in the same way as the

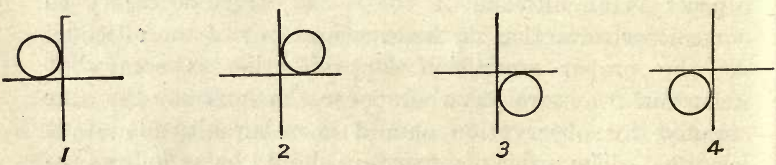


Fig. 32.

stars. The observer must sight to its edge, and in this case, where both horizontal angle and altitude are to be measured, it may be sighted in any one of the four quadrants formed by the cross wires of the telescope. The four different positions in which it may be observed are shown in Fig. 32, the two cross wires at right angles being brought by means of the tangent screws so as to just touch the sun's edge in each case. The centre of the sun's disc is the point considered in all our computations, and this then is the point whose position we seek to determine. Clearly, the centre of the cross wires is midway between the centres of the sun discs in positions 1 and 3, so that the mean of the readings in these two positions should give us the altitude and azimuth of the sun's centre.

Similarly the mean of the readings in positions 2 and 4 will give the position of the sun's centre. A complete set of observations will consist of four observations of the sun in the four positions illustrated. They should be made as follows :—

Take reading of R.M. and clamp horizontal plate.

Turn to the sun and observe altitude and horizontal reading with the sun in quadrant 1 of the cross-wire system.

Then, as quickly as possible, by means of the two tangent screws, bring the sun into quadrant 3 of the cross wires, and again read horizontal angles and altitude.

Turn back to the R.M.

Reverse the face of the instrument and take two more observations in precisely the same way, but this time with the sun in quadrants 2 and 4.

Be careful to note the time of each observation.

During the whole time occupied by the four observations the sun's position will have changed too much for accurate results to be obtained by averaging the measured altitudes and times of the four observations. There should, however, be very little time lost between the first two readings, with the sun in quadrants 1 and 3, and the measured altitudes and times of these two may be averaged together and a computation made for the corresponding azimuth of the referring mark. Similarly, another computation is made, by averaging the readings with the sun in quadrants 2 and 4, from which the azimuth of the referring mark is again determined. Thus we obtain two computed azimuths, one with each face of the instrument, and the average of the two is taken.

The two succeeding observations made without change of face in quadrants 1 and 3 or quadrants 2 and 4 are sometimes a little simplified by what is known as the "run through" method. In this method the observer, after making the first observation, leaves the telescope clamped in vertical arc, and makes the second observation when the sun has just crossed the horizontal wire by moving the vertical wire to the correct position, with the aid of the tangent screw attached to the horizontal circle. The necessity of recording a second set of vertical angles is thus avoided. The objection to this method is that the two observations cannot be made in such quick succession as is possible by the method outlined above, and consequently the error made by taking the average of the two observations is greater.

Very commonly only two observations of the sun are made, and in that case the best procedure is as follows :— 1. Observe the R.M., say, with face L. 2. Observe the sun in, say, quadrant 1 with face L. 3. Reverse face and

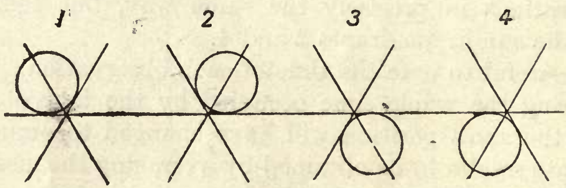


Fig. 32*a*.

observe the sun again as quickly as possible with face R. in quadrant 3. 4. Observe R.M. again with face R. The average of the two observations is then taken as the basis of a single computation.

Should the telescope have its cross wires of the form shown in Fig. 32*a*, the observations will be precisely the same, but the various positions of the sun's image will be as illustrated.

For good work the altitude readings should always be corrected by means of the alidade level, reading the E. and O. ends at each observation.

Computation of Sun's Declination from Nautical Almanac Data.—In the Nautical Almanac the sun's declination is given for both mean and apparent noon at Greenwich, for every day of the year, and also its rate of variation in one hour at Greenwich noon. If the declination is required at, say, 8 hours after Greenwich noon, it will not be accurately found by multiplying the hourly variation by 8 and adding or subtracting the result to the value of the declination at Greenwich noon, because the hourly variation itself is not constant, but changes from hour to hour. The proper plan is to find the mean value of the hourly variation over the interval in question, which in this case will be the value at the middle of the interval —*i.e.*, 4 hours after noon.

EXAMPLE.—*Required the value of the sun's declination at* 9 *hrs.* 20 *min. a.m. on August 2nd,* 1914, *the time being South Australian standard, that of the meridian* 9 *hrs.* 30 *min. E.*

Corresponding astronomical time, .	August 1st, 21 hrs. 20 min.
Corresponding Greenwich time, .	August 1st, 11 hrs. 50 min. p.m.

Hourly variation at G.M.N. on August 2nd,	38·05″
Hourly variation at G.M.N. on August 1st,	37·32″
	0·73″

The half of 11 hrs. 50 min. is very nearly 6 hrs. Therefore, the average hourly variation is

$$37{\cdot}32 + \frac{0{\cdot}73}{4} = 37{\cdot}5$$

$$11{\cdot}83 \text{ hrs.} \times 37{\cdot}5 = 443{\cdot}6'' = 7'\ 23{\cdot}6''.$$

Sun's declination at G.M.N., August 1st, 18° 10′ 50·4″ N., and it is decreasing at this time of the year.

∴ Sun's declination at given time, . 18° 03′ 26·8″ N.

Corrections to Sun Observations.—We have already seen in Chapter VII. that the sun is one of those bodies the observed altitude of which must be corrected for parallax. It must also be corrected for Refraction, as shown in the same chapter.

Either from want of time or through the intervention of clouds the surveyor may be unable to complete the series of four observations, but any single observation will enable him to determine the position of the sun's centre by making proper allowance for the sun's semi-diameter, the value of which is tabulated in the Nautical Almanac.

There is no difficulty with regard to the determination of the altitude of the sun's centre from one observation, as the semi-diameter has simply to be added on or subtracted as the case may be. If, for instance, with a reversing telescope the sun is observed in quadrant 1, it will mean that we are actually sighting the upper edge of the sun, and the measured altitude will have to be reduced by the value of the semi-diameter given in the Nautical Almanac.

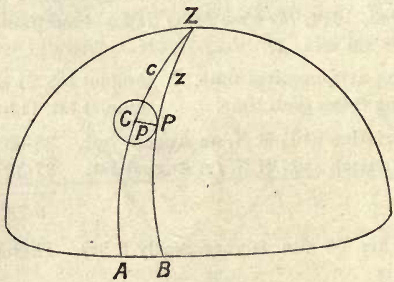


Fig. 33.

But with the observations for azimuth the matter is not quite so simple. Thus, in Fig. 33, if C denotes the centre of the sun's disc, Z the zenith, Z C A the vertical trace on the celestial sphere passing through C and the zenith, Z P B the vertical plane just touching the edge of the sun's disc, then the error in azimuth made by sighting the edge instead of the centre of the sun's disc is the angle C Z P. But in the right-angled triangle Z C P we have

$$\sin CP = \sin CZ \,.\, \sin CZP.$$

Now C P is an angle of about 15 minutes, and its circular measure differs from its sine by 1 only in the seventh place of decimals. Consequently, we may write

$$\text{C P} = \sin \text{C Z} \times \text{C Z P}$$

$$\therefore \qquad \text{C Z P} = \text{C P} \times \operatorname{cosec} \text{C Z},$$

or correction in azimuth

$$= \text{semi-dia.} \times \text{sec. altitude sun's centre.}$$

The Effect of an Error in Latitude upon the Calculated Azimuth.—Referring to Fig. 31, we shall determine the effect of an error in latitude if, in the spherical triangle P Z S, we investigate the effect upon the angle Z of a small change in c, the sides p and z remaining constant.

Let x be the change produced in Z by a small alteration y in c. Then

$$\cos p = \cos c \cos z + \sin c \,.\, \sin z \cos \text{Z} \text{ (formula (2) Chap. I.)}$$

and $\cos p = \cos (c + y) \cos z + \sin (c + y) \sin z \cos (\text{Z} + x)$.

Subtracting these two equations, writing x and y in place of $\sin x$ and $\sin y$, and unity in place of $\cos x$ and $\cos y$, we get

$$\begin{aligned} 0 &= \cos z \,.\, y \,.\, \sin c + \sin z \sin c \cos \text{Z} - \sin z (\sin c \\ &\qquad + y \cos c)(\cos \text{Z} - x \,.\, \sin \text{Z}) \\ &= \cos z \,.\, y \sin c - y \sin z \,.\, \cos c \cos \text{Z} \\ &\qquad + x \,.\, \sin z \,.\, \sin c \,.\, \sin \text{Z}, \end{aligned}$$

neglecting the term involving the product of x and y.

$$\therefore \quad x = \frac{-\cos z \,.\, \sin c + \sin z \cos c \,.\, \cos \text{Z}}{\sin z \sin c \sin \text{Z}} y$$

$$= \frac{-\cot \text{P} \,.\, \sin \text{Z}}{\sin c \sin \text{Z}} \,.\, y \text{ (by formula (3) of Chap. I.)}$$

$$= \frac{-\cot \text{P}}{\sin c} \,.\, y.$$

P is, of course, the hour angle, and we thus have a simple formula for computing the error in azimuth produced by a given error in latitude at any given time of the day. Clearly, when P is very small—that is to say, at times near to noon—cot P is very great, and the error produced by a defective knowledge of the latitude is much increased.

In Fig. 34 a curve is drawn showing the error in azimuth produced by an error of one second in the latitude, at different hours of the day in latitude 40°. It is really a curve of tangents, and it will be seen that the error is very much greater at or near noon than at any other time. The

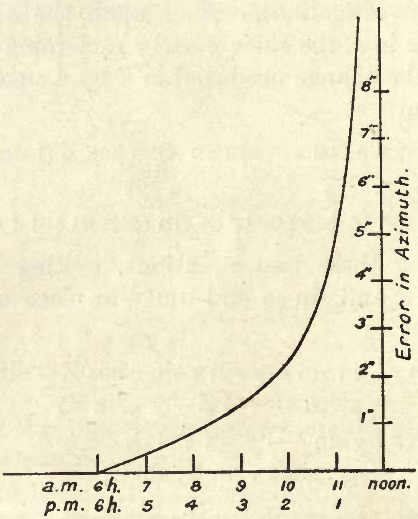


Fig. 34.—Error in Azimuth for Extra Meridian Observation of the Sun, corresponding to error of one second in Latitude, at different hours of the day in Latitude 40°.

error is least at 6 a.m. or 6 p.m. With increase in the latitude of the place of observation the error would be greater still, becoming very great for latitudes near the pole.

The Effect of an Error in the Sun's Declination upon the Calculated Azimuth.—If a slight alteration y is made in the value of p (Fig. 31), c and z remaining constant, then it may be shown in a similar manner to that of the work just preceding that

$$x = \operatorname{cosec} c \operatorname{cosec} P \,.\, y,$$

where x is the corresponding change made in the azimuth Z. The establishment of this formula we will leave as an exercise for the student.

In Fig. 35 a curve is drawn showing the error in azimuth produced by an error of 1 second in the declination at different hours of the day at a place in latitude 40°.

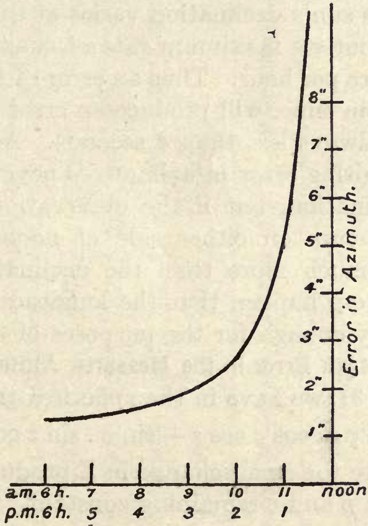


Fig. 35.—Error in Azimuth for Extra Meridian Observation of the Sun, corresponding to error of one second in Declination, at different hours of the day in Latitude 40°.

Again the error is very great near mid-day, and is least at 6 a.m. and at 6 p.m. As in the previous case, with increase in latitude of the place of observation the error also increases, becoming so great in latitudes near the pole that the method would be quite unreliable in arctic or antarctic regions.

It will be noticed that the two errors we have just discussed are of opposite signs, so that, if the declination and latitude are both too large, the errors tend to neutralise one another.

The Effect of an Error in the Longitude of the Place of Observation.—An error in longitude will produce an error in the computed Greenwich time at the instant of observation, and this in turn will produce an error in the calculated declination. An error of 1° in longitude will produce an error of 4 minutes in time. Now the rate of change of the sun's declination varies at different seasons of the year, but its maximum rate of change is less than 1 minute of arc per hour. Thus an error of 1° in longitude, or 4 minutes in time, will produce an error in the declination that is always less than 4 seconds. As we have just seen, the resulting error in azimuth is never less than the error in declination, but if the observation is not made within two hours on either side of noon, the azimuth error is not much more than the declination error. It will thus seldom happen that the longitude is not known approximately enough for the purposes of the surveyor.

The Effect of an Error in the Measured Altitude.—Referring again to Fig. 31, we have in the spherical triangle S P Z

$$\cos p = \cos c \cos z + \sin c \,.\, \sin z \cos Z.$$

Let x denote the small change in Z produced by a small change y in z, p and c remaining constant. Then

$$\cos p = \cos c \,.\, \cos (y + z) + \sin c \,.\, \sin (y + z) \cos (x + Z).$$

Subtracting and simplifying these equations, regarding x and y as small quantities, we finally arrive at the result

$$x = - \cot S \operatorname{cosec} z \,.\, y.$$

Thus x will be infinitely great when $S = o$ or 180°, which is the case when the sun is on the meridian. And, again, we arrive at the result that the resulting error in azimuth is very great if the observation is made near noon, but is small if S is anywhere near 90°.

EXAMPLE OF EXTRA MERIDIAN OBSERVATIONS OF THE SUN FOR AZIMUTH.

Date, ...	25th June, 1914.	Barometer, . . .	29·8 inches.
Place, ...	Adelaide.	Thermometer, . .	62° Fahr.
Latitude, ...	34° 55′ 38·5″.	Longitude, .	9 hrs. 14 min. 20·3 sec. E.

Object.	Face.	Standard Time of Observation.	Horizontal Circle.			Vertical Circle.		
			A.	B.	Mean.	A.	B.	Mean.
R.M.	R		360°	180°	360°			
$\underline{\odot}\vert$	R	H. M. S. 2 56 00	203° 2′ 20″	23° 2′ 20″	203° 2′ 20″	20° 26′ 20″	20° 26′ 00″	20° 26′ 10″
$\vert\overline{\odot}$	R	2 57 30	203° 6′ 20″	23° 6′ 20″	203° 6′ 20″	20° 45′ 20″	20° 45′ 10″	20° 45′ 15″
R.M.	R		360°	180°	360°			
Mean angle between R.M. and sun = 156° 55′ 40″.						Mean altitude of sun, 20° 35′ 42·5″.		
R.M.	L		180°	360°	360°			
$\vert\underline{\odot}$	L	3 01 13	22° 33′	202° 33′	202° 33′	19° 53′	19° 53′	19° 53′
$\overline{\odot}\vert$	L	3 01 50	21° 50′	201° 50′	201° 50′	20° 11′	20° 11′	20° 11′
R.M.	L		180°	360°	360°			

Mean angle between R.M. and sun = 157° 48′ 30″. Mean altitude of sun, 20° 02′.

The Best Time for Extra-Meridian Observations. — The preceding discussions all point to the desirableness of making the observations upon the sun as far away from noon as possible. But if we observe it when too low down

COMPUTATION.

	F.R.	F.L.
Standard time of observation, June 25th, .	2 hrs. 56 min. 45 sec.	3 hrs 01 min. 31 sec.
Longitude East, . .	9 hrs. 30 min.	9 hrs. 30 min.
Corresponding G.M.T., June 24th, . .	17 hrs. 26 min. 45 sec.	17 hrs. 31 min. 31 sec
Sun's declination at G.M.N., . . .	23° 26′ 09·4″	23° 26′ 09·4″
Variation since G.M.N.,	46·4″	46·6″
Sun's declination when observed, . .	23° 25′ 23″	23° 25′ 22·8″
Observed altitude, .	20° 35′ 42·5″	20° 02′ 00″
Refraction and parallax,	2′ 22″	2′ 27″
Corrected altitude, .	20° 33′ 20·5″	19° 59′ 33″
Zenith distance = z, .	69° 26′ 39·5″	70° 00′ 27″
Sun's polar distance = p,	113° 25′ 23″	113° 25′ 22·8″
Co-latitude = c, . .	55° 04′ 21·5″	55° 04′ 21·5″
$2s$, ...	237° 56′ 24″	238° 30′ 11·3″
s, ...	118° 58′ 12″	119° 15′ 05·6″
$s - p$, . . .	5° 32′ 49″	5° 49′ 42·8″
L sin s, . . .	9·9419452	9·9407569
L sin $(s - p)$, . .	8·9852526	9·0066890
L cosec z, . . .	10·0285705	10·0269935
L cosec c, . . .	10·0862505	10·0862505
L $\cos^2 \frac{1}{2}$ Z, . . .	19·0420188	19·0606899
L cos $\frac{1}{2}$ Z, . . .	9·5210094	9·5303449
$\frac{1}{2}$ Z, ...	70° 36′ 57″	70° 10′ 38″
Z (from South), . .	141° 13′ 54″	140° 21′ 16″
Bearing of sun, . .	321° 13′ 54″	320° 21′ 16″
Angle between sun and R.M., ...	156° 55′ 40″	157° 48′ 30″
Azimuth of R.M., .	118° 09′ 34″	118° 09′ 46″

in the heavens, the refraction becomes a very uncertain quantity, and consequently it is impossible to measure the altitude with precision. For this reason it is generally considered inadvisable to make the observation with the sun at a lower altitude than about 15°. With this limitation it is desirable, in order to minimise the effects of errors in altitude, latitude, and declination, to make the observation as far from noon as possible. So that if the readings are made in the morning, they should be made as soon as possible after the sun has reached an altitude of 15°. Similar remarks will apply to the stars, which should be observed as far away from the meridian as possible, so long as they are at an altitude of at least 15° above the horizon.

Fourth Method—Time Observations upon a Close Circumpolar Star.—The method about to be described is the one chiefly adopted on geodetic surveys where the highest attainable degree of accuracy is desired. The observations consist in measuring a series of angles between a close circumpolar star and the R.M., noting the time at which each pointing is made to the star. No altitudes need be measured, and as the time may be measured with sufficient precision by means of a chronometer, the method is simple, as well as capable of great accuracy. In the Northern Hemisphere the star α Ursæ Minoris (Polaris) is a very convenient one for the purpose. Being a star of the second magnitude, it can be readily found, and it is within about 1° 10′ of the N. Pole. λ Ursæ Minoris is within 1° of the Pole, but is a much fainter star, being of magnitude 6·6. Other suitable Northern circumpolar stars are 51 Cephei (Mag. 5·2) and δ Ursæ Minoris (Mag. 4·4). In the Southern Hemisphere, unfortunately, there are no stars near the pole sufficiently bright to be readily picked out without first of all calculating their positions. The best star for the purpose is σ Octantis, which is within 46′ of the S. Pole. It is, however, of magnitude 5·5, and in order to pick up the star it is necessary to know beforehand the approximate bearing of the R.M. This may be found from a daylight observation by one of the methods previously described.

In Fig. 36, let P be the celestial pole around which circulates in a small circle the circumpolar star S. Let Z be the zenith. Then in the spherical triangle Z P S, $ZP = c =$ co-latitude, $PS = p =$ polar distance of star, $ZPS = t =$ hour angle of star. $PZS = Z =$ azimuth angle of star.

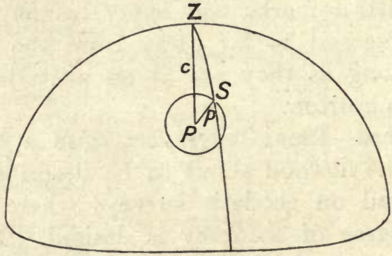


Fig. 36.

From formula (3) in Chapter I.

$$\cot p \sin c = \cot Z \sin t + \cos c \cos t,$$

$$\therefore \qquad \cot Z = \frac{\cot p \sin c - \cos c \cos t}{\sin t}$$

$$= \frac{\sin (c - x) \cot t}{\sin x}, \quad . \quad . \quad . \quad (1)$$

where $\tan x = \tan p \cos t$.

The hour angle t, in time, is found by taking the difference between the R.A. of the star, which is the sidereal time when the star is on the meridian, and the sidereal time at the moment of observation. To determine this we must know both the local mean time and the longitude of the place. Thus, we require to know the R.A. and declination of the star and also the latitude and longitude of the place of observation.

For the most accurate work the striding level should be used to determine the error in the measured azimuth of the star owing to any defect in the levelling of the transverse axis of the telescope. This will produce an appreciable effect upon the azimuth of the star owing to its altitude, but as the R.M. will usually be near the horizon, it will not as a rule be necessary to apply any correction on this account to the reading taken to it. If, however, the R.M. should be at a considerable altitude, it would be necessary to read the striding level both when the telescope is pointed to the star and when it is directed to the R.M.

The series of observations necessary may be arranged in several different ways. The following is the programme recommended by the U.S. Coast and Geodetic Survey :—

1. Point twice upon the R.M. and read the verniers of the horizontal circle at each pointing, the instrument being F.L.

2. Read twice on the star with F.L., noting at each pointing the exact time, the reading of each end of the striding level, and the readings of the horizontal circle.

3. Read twice on the star with F.R., the instrument being reversed, noting the time and bubble readings as before.

4. Read twice upon the R.M. with F.R.

According to this programme the striding level is left with the same ends on the same pivots throughout the observations.

The programme suggested in the handbook of instructions for Western Australian Surveyors is as follows :—

1. Set the instrument to zero, point to R.M., and read the circle.

2. Intersect star and take the time.

3. Read the striding level and reverse it.

4. Read the circle.

5. Intersect star again and take the time.

6. Read the striding level.
7. Read the circle.
8. Point to R.M. and read the circle.

In turning back to R.M. the instrument is moved in the opposite direction. The instrument is now reversed, the setting on the R.M. increased by 22° 30′, and the operation repeated until angles have been read all round the circle.

A series of observations having been taken by one of these systems, the hour angle of the star and the corresponding azimuth will, of course, be different for each pointing. Each separate observation will give us the azimuth of the R.M., and we wish to get the mean or average of these determinations. We may compute the azimuth of the star for each pointing separately by means of formula (1), deducing from each computation the azimuth of the R.M., and then take the average of the different results. This is the simplest procedure, involving no mathematical difficulties, and when only a few observations have been taken this is the best plan to adopt. But when there are a number of observations the calculations may be lessened by computing the azimuth corresponding to the mean of the several hour angles. This would not be the same as the mean of the different azimuths, but the latter may be derived from the former by applying a correction known as the "curvature correction." In the case of a close circumpolar star, and a series of observations not extending over about half an hour, the curvature correction is given by the formula

$$\text{Correction} = \tan A \frac{1}{n} \Sigma \frac{2 \sin^2 \frac{1}{2} (T - T_0)}{\sin 1''},$$

where A = computed azimuth of star at the mean hour angle of n pointings,

T_0 = mean of the n hour angles.

T = any one of the separate hour angles.

The establishment of this correction is rather beyond the mathematical scope of this work.

The true mean azimuth always lies nearer the meridian than the azimuth corresponding to the mean hour angle.

The expression $\frac{2 \sin^2 \frac{1}{2} (T - T_0)}{\sin 1''}$, which is usually denoted by m, has to be evaluated for each observation, and as the same form also enters into the computation of circum-meridian observations for latitude, tables have been computed, available in various works, such as Chauvenet's Astronomy and Trigonometrical Surveying by Major Close, in which the values are tabulated for different values of $T - T_0$. The use of such tables greatly facilitates the computation, as the curvature correction is then found by adding the different values taken from the tables, dividing the sum by n and multiplying by tan A. A table giving the values of m at intervals of 10 seconds of time up to 19 minutes is given at the end of Chapter IX.

Circum-Elongation Observation for Azimuth.

The following account is extracted from a paper by the author published in the *Transactions of the Royal Society of South Australia*, vol. xxxix., 1915. The mathematics involved is rather more advanced than that in the rest of this work, but the method is of sufficient importance to make it desirable to insert it :—

On account of its convenience and comparative simplicity, the observation of a circumpolar star at elongation is, amongst surveyors, the favourite star observation for the determination of a true azimuth. The great disadvantage of the method is that only one observation can be made with the star actually at elongation, and there is thus no opportunity to eliminate instrumental errors in the same way as may be done, when a series of observations of the same star are made, by taking half the readings with the instrument reversed. As a rule the motion of the star in azimuth is so slow, when near to elongation, that with an ordinary transit theodolite two observations can be made and treated as though the star were actually at elongation without introducing an error sufficient to be measured by the instrument. But a much higher degree of accuracy is possible with the method if a series of half a dozen observations are made on each side of elongation, and the object of the present paper is to discuss the convenient reduction of such a series of observations. For the reduction of a similar set of observations made upon a *close* circumpolar star there is a well-known method that is particularly applicable to the Pole star of the Northern Hemisphere. Unfortunately in the Southern Hemisphere the close circumpolar stars are very faint and not easy to work with. σ Octantis has a polar distance between 46′ and 47′, but its magnitude is $5\frac{1}{2}$, so that it is not readily picked out by the surveyor. The bright southern stars that are most convenient for the determination have commonly a polar distance of about 30°, and to these the formula for close circumpolar stars cannot always be applied without introducing appreciable error.

Two methods are possible for a series of observations made before and after elongation. We may read the verniers of the horizontal circle and note the time at each observation, or we may read the horizontal circle and also the altitude of the star at each observation. The former method is preferable, provided that the surveyor has the correct local time, as errors due to a defective knowledge of atmospheric refraction are not then introduced. The latter method, however, involves no knowledge of the time, and is much more convenient when the observations have to be carried out single-handed. In both cases the azimuth of the star at each observation is corrected by the appropriate formula to give the azimuth of the star at elongation, so that practically we obtain a series of observations at elongation instead of only one.

Notation.

The following abbreviations will be used throughout :—

z denotes the zenith distance of the star in any position.
p ,, polar distance of the star.
A ,, horizontal angle between star and pole.
l ,, latitude of place of observation.
c ,, co-latitude of place of observation.
h ,, hour angle of the star in angular measure.
t ,, value of hour angle expressed in sidereal time.

z_0, A_0, h_0, and t_0 denote the values of z, A, h, and t respectively when the star is at elongation.

First Method—Horizontal Angle and Time being Noted at each Observation.—In the spherical triangle having the star, the celestial pole, and the zenith as its angular points we have the following fundamental relations :—

$$\cos A \sin z = \cos p \sin c - \cos c \sin p \cos h, \quad . \quad (1)$$

$$\sin A \sin z = \sin p \sin h, \quad . \quad . \quad . \quad . \quad (2)$$

and from the corresponding right-angled triangle when the star is at elongation

$$\sin A_0 = \frac{\sin p}{\sin c} = \frac{\cos p \cos h_0}{\cos c} . \quad . \quad . \quad (3)$$

$$\cos A_0 = \cos p \sin h_0 . \quad . \quad . \quad . \quad (4)$$

(1) × (3) − (2) × (4) gives

$$\sin z \sin (A_0 - A) = \cos p \sin p \, 2 \sin^2 \tfrac{1}{2} (h_0 - h). \quad . \quad (5)$$

This is an exact equation, but is unsuitable as it stands for use in reduction of observations.

Putting $\dfrac{\sin p}{\sin z} = \dfrac{\sin A}{\sin h}$, (5) may be written

$$\frac{\sin (A_0 - A)}{\sin A} = \cos p \,\frac{2 \sin^2 \frac{1}{2} (h_0 - h)}{\sin h}$$

or, writing
$$y = \cos p \,\frac{2 \sin^2 \frac{1}{2} (h_0 - h)}{\sin h}$$

$$\sin A_0 \cot A - \cos A_0 = y.$$

A_0 is constant, and, therefore, A may be regarded as a function of y.

Differentiating, we have

$$- \sin A_0 \frac{1}{\sin^2 A} \frac{d A}{d y} - 1,$$

and
$$- \sin A_0 \frac{d^2 A}{d y^2} = 2 \sin A \cos A \frac{d A}{d y}.$$

Therefore, when $y = o$

$$\frac{d A}{d y} = - \sin A_0 \text{ and } \frac{d^2 A}{d y^2} = \sin 2 A_0,$$

and consequently, by Taylor's Theorem

$$A = A_0 - \sin A_0 \frac{\cos p \, 2 \sin^2 \frac{1}{2} (h_0 - h)}{\sin h \sin 1''}$$
$$+ \sin 2 A_0 \frac{\cos^2 p \, 2 \sin^4 \frac{1}{2} (h_0 - h)}{\sin^2 h \sin 1''},$$

provided that $A_0 - A$ is measured in seconds of arc.

This is a convenient converging series for the determination of the difference between A and A_0, in which the terms diminish so rapidly that in all ordinary work it is not necessary to take into account any term except the first. Thus, if the observations are made at a place in latitude 30°, on a star with a polar distance of 30°, and are continued for fifteen minutes of time on each side of elongation, the extreme value of $h - h_0 = 3° 45'$. The corresponding value of the first term in the series then works out at 229″, or 3′ 49″, and that of the second term at less than $\frac{1}{2}''$. If $t - t_0 = 30$ minutes, or $h - h_0 = 7° 30'$, then under the same conditions the first term $= 902''$ and the second term only $5\frac{1}{2}''$. With the same polar distance and in the same latitude, the limiting value for $t - t_0$, in order that the second term may not be greater than 1″, is about 19 minutes. On repeating the calculations for a place in latitude 20°, and again for a place in latitude 40°, it is found that in neither case does the limiting value of $t - t_0$ differ by more than a minute from the value previously found if the second term in the series is to be less than 1″.

It thus appears that, even if the mathematical reduction of each single observation is to be correct within 1″ of arc, it is sufficient to use only the first term of the series if the observations extend over a period of about 19 minutes on each side of the elongation. The average of the whole series may be correct within this limit, even if the time extends over a considerably longer period, because the error in reduction will exceed 1″ only in the case of the extreme observations.

A further considerable simplification would be made in the reduction if it were possible to treat the denominator as constant and write $\sin h_0$ instead of $\sin h$. With any single observation the error made, if this is done, may be considerable. For instance, at a place in latitude 30°, if $p = 30°$, for an observation made 15 minutes before elongation, the difference made in the value of the second term, when $\sin h_0$ is written in the denominator instead of $\sin h$, is about 5″, whilst for an observation made 30 minutes before elongation the difference is about 35″. But, if we have a series of fairly well-balanced observations made both before and after elongation, the values of h range fairly evenly on each side of h_0, and on averaging up the set there will be very little difference whether we use h or h_0, the difference being generally of the order of $1''$. So that in such a case it is usually quite sufficient for the surveyor to use h_0 instead of h. We may then make a further slight simplification by putting

$$\frac{\sin A_0 \cos p}{\sin h_0} = \tan A_0 \cos^2 p.$$

Practical Computation.—We therefore conclude that, for the ordinary work of the surveyor, a series of well-balanced observations extending to about half an hour on each side of elongation on any circumpolar star may be reduced to a series of observations at elongation by the formula

$$A_0 - A = \tan A_0 \cos^2 p \; 2 \frac{\sin^2 \frac{1}{2}(h_0 - h)}{\sin 1''}, \qquad (6)$$

in which $A_0 - A$ is given in seconds of arc.

If, however, only one or two observations are to be reduced, as may be the case if the star at elongation has been obscured by clouds, or the observations are badly balanced and have been made mostly on one side of elongation, or if the greatest possible degree of accuracy is required in the computations, the formula used should be

$$A_0 - A = \sin A_0 \frac{\cos p \; 2 \sin^2 \frac{1}{2}(h_0 - h)}{\sin h \sin 1''}. \qquad (7)$$

This form may be obtained directly from (5) by considering $A_0 - A$ as a small angle so that the sine may be written equal to its circular measure.

If it is required to make the computation within $1''$, then, for observations more than 18 minutes from elongation, the value of $A_0 - A$ given by formula (7) should be corrected by being decreased by the amount

$$\sin 2 A_0 \frac{\cos^2 p \; 2 \sin^4 \frac{1}{2}(h_0 - h)}{\sin^2 h \sin 1''}. \qquad (8)$$

As the expression $\dfrac{2 \sin^2 \frac{1}{2} (h_0 - h)}{\sin 1''}$ has to be evaluated in the reduction of circum-meridian observations for latitude, tables of the value of the expression and its logarithm have been prepared, and are available in Chauvenet's Astronomy, Close's Astronomical Surveying, and other works. An abbreviated table is given at the end of Chap. IX. Similar tables for $\dfrac{2 \sin^4 \frac{1}{2} (h_0 - h)}{\sin 1''}$ are also available. The computation by any one of these formulæ is much facilitated by the use of these tables. Five-figure logs are sufficient.

Writing $\tan A_0 \cos^2 p = B$, $m = \dfrac{2 \sin^2 \frac{1}{2} (h_0 - h)}{\sin 1''}$.

(6) becomes

$$A_0 - A = B\,m, \text{ where } B \text{ is a constant.}$$

Thus for each observation we get $A_0 = A + B\,m$, and, averaging the whole series,

Mean value of A_0 = mean value of $A + B \times$ mean value of m.

Therefore, mean angle between R.M. and star at elongation = mean observed angle between R.M. and star $\pm B \times$ mean value of m.

Example.—In the following example the method is applied to the reduction of a series of observations taken by Mr. Calder, surveyor, upon Canopus near elongation :—

Star observed—Canopus.
Place—Rendelsham, South Australia.
Right Ascension—6 hrs. 22 min. 06 sec.
Latitude—37° 32′ 40″ S.
Declination—52° 38′ 43″ S.
Longitude—9 hrs. 20 min. 40 sec. E.
Date—December 9th, 1914.
Standard Meridian—9 hrs. 30 min. E

Computed Values.

Standard time at elongation—9 hrs. 45 min. 32 sec. p.m.

$$A_0 = 49° \ 55' \ 44''$$
$$h_0 = 54° \ 04' \ 50''$$

Face.	Object.	Mean Vernier Readings on Horizontal Circle.	Standard Time of Observation.			Interval of Mean Time between Observation and Elongation.		Corresponding Interval in Sidereal Time.	
			H.	M.	S.	min.	sec.	min.	sec.
R	R.M.	360°							
R	Star	83° 16′ 00″	9	32	44	12	48	12	50
L	Star	83° 15′ 15″	9	34	37	10	55	10	57
L	R.M.	360°							
L	Star	83° 13′ 45″	9	38	25	7	07	7	08
R	Star	83° 13′ 00″	9	40	15	5	17	5	18
R	R.M.	360°							
R	Star	83° 12′ 15″	9	43	05	2	27	2	27
L	Star	83° 12′ 45″	9	45	11		21		21
L	R.M.	360°							
L	Star	83° 12′ 15″	9	48	40	3	08	3	09
R	Star	83° 13′ 15″	9	50	55	5	23	5	24
R	R.M.	360°							
R	Star	83° 16′ 45″	9	58	17	12	45	12	47
L	Star	83° 18′ 15″	10	01	00	15	28	15	31
L	R.M.	360°							

Mean observed angle between star and R.M., 83° 14′ 21″.

Solving by means of (6), we obtain from the tables :—

$t_0 - t$		m
min.	sec.	
12	50	323·3″
10	57	235·4″
7	08	99·9″
5	18	55·1″
2	27	11·8″
	21	0·2″
3	09	19·5″
5	24	57·2″
12	47	320·8″
15	31	472·6″
		10) 1,595·8
		159·6

Mean value of m

$$\begin{aligned} \log \tan A_0 &= 10 \cdot 07509 \\ \log \cos^2 p &= 9 \cdot 80062 \\ \log 159 \cdot 6 &= 2 \cdot 20303 \\ \hline \log 120 &= 2 \cdot 07874 \end{aligned}$$

$\therefore\ B m = 120'' = 2'$

$\therefore$ Mean value of angle between R.M. and star at elongation $= 83^\circ\ 14'\ 21'' - 2'\ 0'' = 83^\circ\ 12'\ 21''$

The computation by means of the more accurate formula (7) is rather longer. In this case we write

$$B = \sin A_0 \cos p \text{ and } m = \frac{2 \sin^2 \frac{1}{2} (h_0 - h)}{\sin h \sin 1''},$$

and work on the same lines as before. To illustrate the method the computation in this case is also worked out as follows :—

$t_0 - t$.	$h_0 - h$.	h.	$\log \frac{2 \sin^2 \frac{1}{2}(h_0 - h)}{\sin 1''}$	log sin h.	log m = difference of two preceding columns.	m.
min. sec.						
12 50	3° 12′ 30″	57° 17′ 20″	12·50960	9·92501	2·58459	394·2
10 57	2° 44′ 15″	56° 49′ 05″	12·37178	9·92269	2·44909	281·5
7 08	1° 47′ 00″	55° 51′ 50″	11·99958	9·91788	2·08170	120·7
5 18	1° 19′ 30″	55° 24′ 20″	11·74157	9·91550	1·82607	67·0
2 27	36′ 45″	54° 41′ 35″	11·07136	9·91173	1·15963	14·4
21	5′ 15″	54° 10′ 05″	9·38117	9·90888	$\bar{1}$·47229	0·3
3 09	47′ 15″	53° 17′ 35″	11·28965	9·90401	1·38564	24·3
5 24	1° 21′ 00″	52° 43′ 50″	11·75780	9·90080	1·85700	71·9
12 47	3° 11′ 45″	50° 53′ 05″	12·50621	9·88979	2·61642	413·4
15 31	3° 52′ 45″	50° 12′ 05″	12·67446	9·88553	2·78893	615·1
						10) 1992·8
			Mean value of m,	.	.	. 199

$$\begin{aligned} \log \cos p &= 9 \cdot 90031 \\ \log \sin A_0 &= 9 \cdot 88380 \\ \log 199 &= 2 \cdot 29885 \\ \hline \log 121 &= 2 \cdot 08296 \end{aligned}$$

$\therefore\ B m = 121'' = 2'\ 01''$

$\therefore$ Mean value of angle between R.M. and star at elongation $= 83^\circ\ 14'\ 21'' - 2'\ 01'' = 83^\circ\ 12'\ 20''$

The difference between the results of the two calculations is so small that clearly the more simple approximate method is quite sufficient for the surveyor. If the computation be made for the last four observations only, the difference between the results of the two methods amounts to 8″, and for the last observation alone the difference is 19″. For the surveyor it is only necessary to use the more accurate method of calculation for unbalanced observations at a considerable time from elongation.

It may be proved that, *provided the observations extend evenly over an equal time on each side of elongation*, there is no need for the surveyor to know the local time with great precision, an error of 1 minute in the time producing an error of only about 1″ in the azimuth.

But if the observations do not extend on each side of elongation the case is different, and a more accurate knowledge of the time is essential.

Second Method—Horizontal Angle and Altitude being Noted at each Observation.—With the same notation as before, the star being in any position, we have

$$\cos p = \cos c \cos z + \sin c \sin z \cos A.$$

Writing $x = z - z_0$, this becomes

$$\cos p = \cos c \cos (z_0 + x) + \sin c \sin (z_0 + x) \cos A.$$

p, c, and z_0 being constants, this equation gives A as an implicit function of x.

Differentiating the equation three times in succession, the work being rather long but quite straightforward, we find that when $x = o$

$$\frac{d\,A}{d\,x} = o,$$

$$\frac{d^2 A}{d\,x^2} = -\frac{\cot p}{\sin z_0}$$

$$\frac{d^3 A}{d\,x^3} = \frac{3 \cot p \cos z_0}{\sin^2 z_0}.$$

Therefore, by Taylor's Theorem

$$A = A_0 - \frac{\cot p}{\sin z_0}\,\frac{(z - z_0)^2}{2}\sin 1''$$
$$+ \frac{\cot p \cos z_0}{\sin^2 z_0}\,\frac{(z - z_0)^3}{2}\sin^2 1'' \quad . \quad . \quad (9)$$

provided that $A_0 - A$ and $z - z_0$ are expressed in seconds of arc.

To get some idea of the relative values of the terms in this series, we find, if the star observed has a polar distance of 30° and the latitude is also 30°, then $z_0 = 54°\ 44'\ 09''$, and if $z - z_0 = 1°$, the second term works out at 66″ and the last term to 0·8″. If $z - z_0 = 2°$ the values become 264″ and 6″ respectively.

The last term in (9) is equal to

$$\frac{\cos^2 p \cos c}{\sin p\,(\cos^2 p - \cos^2 c)} \times \frac{(z - z_0)^3}{2}\sin^2 1'',$$

and has, therefore, an infinite value if $p = c$, in which case the star passes through the zenith. This is clearly of no practical importance.

The following are the values of the last terms in different latitudes for a star 30° distant from the celestial pole, if $z - z_0 = 1°$:—

Latitude.	Value of Last Term in (9).
50°,	3·5″
40°,	1·5″
30°,	0·8″
20°,	0·4″
10°,	0·2″
0°,	0″

If $z - z_0 = 2°$ the preceding values should be multiplied by 8.

It follows, therefore, that for the ordinary work of the surveyor the correction involved in the last term of the series is quite negligible for observations extending over a range of altitude of 2°, or 1° on each side of elongation, provided that the star does not pass within 10° of the zenith. At places near the equator the observations may clearly extend over a very much greater range of altitude with the same degree of precision.

To determine over what range of time the observations may extend, we find on differentiating the equation

$$\cos z = \cos c \cos p + \sin c \sin p \cos h$$

that $\dfrac{dz}{dh} = \dfrac{\sin c \sin p \sin h}{\sin z} = \sin p$ for a star at elongation.
This $= \frac{1}{2}$, if $p = 30°$.

Thus, the rate of change of altitude at elongation does not depend on the latitude, but simply on the polar distance of the star, and for a star distant 30° from the pole we have

$$dh = 2\,dz.$$

Therefore, if $dz = 1°$, $dh = 120'$ of arc, or 8 minutes of time, the altitude of the star near elongation thus changes by 1° in about 8 minutes. For stars closer to the pole the time taken for the same change of altitude will be greater.

Practical Computation.—We conclude that for a set of observations extending over a range of altitude of about 2°, or 1° on each side of elongation, occupying, in the case of a star with a polar distance of 30°, about 16 minutes of time, it is amply sufficient to use the formula

$$A_0 - A = \frac{\cot p}{\sin z_0} \frac{(z - z_0)^2}{2} \sin 1''. \quad . \quad . \quad (10)$$

It should be noticed that the error made by the use of this formula in the final reduction of a set of observations will be very much less than the error made in the reduction of the single observation furthest from elongation. We have based the stated limitations upon the error made in the reduction of the single observation, so that for a complete set of observations the time occupied may be extended somewhat beyond the limits given above. In low latitudes the observations may extend over a greater range than in high latitudes. In latitude 10°, for instance, the observations may extend over half an hour, and formula (10) will still give the average result of the set of readings correct within less than 1″.

If the range of altitude is too great, or it is desirable to compute $A_0 - A$ with the greatest precision possible, then this value must be *reduced* if $z > z_0$, or *increased* if $z < z_0$, by the amount

$$\frac{\cot p \cos z_0}{\sin^2 z_0} \frac{(z - z_0)^3}{2} \sin^2 1''. \quad . \quad . \quad (11)$$

The computation by means of (10) is somewhat facilitated by making use of the same tables for circum-meridian calculations as have been shown to be suitable for the reduction by the first method. For since $z - z_0$ is a small angle, we have, within the degree of accuracy to which the tables are computed,

$$\frac{(z - z_0)^2}{2} \sin 1'' = \frac{2 \sin^2 \frac{1}{2} (z - z_0)}{\sin 1''},$$

and consequently we can take the value of $\frac{(z - z_0)^2}{2} \sin 1''$ straight from the tables.

Then, writing

$$B = \frac{\cot p}{\sin z_0}, \; m = \frac{(z - z_0)^2}{2} \sin 1'',$$

we get for each observation, just as in the previous method,

$$A_0 = A + B\,m\,;$$

or, angle between R.M. and star at elongation

$$= \text{observed angle between R.M. and star} \pm B\,m.$$

Since B is a constant, we therefore get, on averaging the whole set of observations :—

Mean angle between R.M. and star at elongation
= mean observed angle between R.M. and star
$\pm B \times$ mean value of m.

Whether the + or − sign is to be used depends upon the position of the R.M. and upon which angle between the star and R.M. is measured. It will be obvious in any particular case which sign should be taken.

If the tables for m are not available, then it is better to write

$$B = \frac{\cot p}{\sin z_0} \frac{\sin 1''}{2}, m = (z - z_0)^2$$

and proceed as before, this time computing m for each observation. The use of the tables does not thus really make very much difference.

A defective knowledge of refraction does not seriously affect the accuracy of the work. For even if the altitude is in error by 15″, the resulting error in azimuth is only about three-quarters of a second of arc.

The following example illustrates the method of reduction. It will be seen that the calculations are simple, and the method is undoubtedly capable of much greater accuracy than the ordinary methods of making elongation observations :—

Star observed—α^1 Crucis.
Right Ascension—12 hrs. 21 min. 54 sec.
Declination—62° 37′ 47″ S.
Date—March 5th, 1915.
Place—Burnside.
Latitude—34° 55′ 38″ S.
Longitude—9 hrs. 14 min. 36 sec. E.
Standard Meridian—9 hrs. 30 min. E.

Computed Values.

Standard time at elongation—9 hrs. 13 min. 18 sec. p.m.

$$A_0 = 34° \ 06' \ 25''$$
$$z_0 = 49° \ 51' \ 22''$$

Face.	Object.	Mean Vernier Readings on Horizontal Circle.	Observed Zenith Distance.	z = Observed Zenith Distance Corrected for Refraction.	$z - z_0$.	m.
R	R.M.	360°				
R	Star	76° 56′ 30″	50° 53′ 00″	50° 54′ 10″	62′ 48″	34·36″
R	Star	76° 55′ 30″	50° 26′ 00″	50° 27′ 09″	35′ 47″	11·15″
R	Star	76° 55′ 00″	49° 57′ 15″	49° 58′ 23″	7′ 01″	0·43″
L	Star	76° 55′ 15″	49° 36′ 00″	49° 37′ 07″	14′ 15″	1·77″
L	Star	76° 55′ 45″	49° 13′ 45″	49° 14′ 51″	36′ 31″	11·63″
L	Star	76° 57′ 00″	48° 38′ 30″	48° 39′ 34″	71′ 48″	44·92″
L	R.M.	360°				

6) 104·26

Mean value of m, . . . 17·38″

Mean observed angle between star and R.M. = 76° 55′ 50″

$$B = \frac{\cot p}{\sin z_0} = \frac{\tan 62° \ 37' \ 47''}{\sin 49° \ 51' \ 22''} = 2{\cdot}527.$$

Therefore, mean value of angle between R.M. and star at elongation

$$= 76° \ 55' \ 50'' - 2{\cdot}527 \times 17{\cdot}38''$$
$$= 76° \ 55' \ 50'' - 44''$$
$$= 76° \ 55' \ 06''$$

Examples.

1. At a place in latitude 30° N., prove that the azimuth of a circumpolar star having a declination of 80° N. when at Eastern elongation is 11° 34′ 00·8″, and that the hour angle of the star is then 84° 9′ 25·3″.

Find the time taken for the azimuth to decrease by 5″.

Ans. 3 min. 34 sec.

2. At a place in latitude 30° S., prove that the azimuth of a circumpolar star having a declination of 60° S. when at Western elongation is 215° 15′ 51·8″, and that the hour angle of the star is then 70° 31′ 43·6″.

Find the time that elapses before the azimuth is diminished by 5″.

Ans. 2 min. 11 sec.

3. In latitude 37° S., the sun's declination being 14° S., show that at 9 a.m. the sun's azimuth is 72° 14′ 39″.

4. Compute the azimuth of a star having a declination of 75° S. when at Eastern elongation, at a place in latitude 30° S.

Ans. 162° 36′ 39·4″.

5. Demonstrate that if two circumpolar stars A and B are in the same vertical at some instant on the East of the meridian, A being above B, they will later be simultaneously on the vertical making the same angle on the West of the meridian, B being then above A.

6. At Greenwich noon, June 1st, 1914, the declination of the sun is 21° 58′ 52·9″ N., the variation in one hour being 20·96″. At noon on June 2nd the declination is 22° 07′ 04·4″, the variation in one hour being 20·00″. Find the sun's declination when the local time at a place in longitude 50° W. is June 1st, 1914, 4 p.m.

Ans. 22° 01′ 25·5″.

7. The corrected observed zenith distance of the sun on the afternoon of March 17th at a place in latitude 34° 56′ S. is 62° 19′. If the sun's declination is 1° 28′ S., compute its azimuth, to the nearest minute of arc, at the time of observation.

Ans. 289° 20′.

8. At a place in latitude 41° 12′ 40″ S. and longitude 11 hrs. 39 min. 34 sec. E. on the evening of the 15th January, 1913 (with the object of checking a traverse bearing), the altitude and bearing of a second magnitude star were observed through a break in the clouds. It was necessary to compute the approximate R.A. and dec. of the star to identify it in the catalogue, in order to obtain the precise elements for the calculation. From the following data, find the star's R.A. and dec. :—

Star's true altitude, . . .	43° 52′ 34″
Bearing corrected for convergence,	131° 3′ 14″
Sidereal time G.M.N., 15th January, ...	19 hrs. 37 min. 19 sec.
N.Z. standard mean time (11 hrs. 30 min. E.), . . .	8 hrs. 20 min. 51 sec.

Ans. R.A. = 8 hrs. 18 min. 54 sec.
Dec. = 54° 22′ 11″.

9. Determine the difference of azimuth of the sun at its rising in mid-winter and mid-summer, also the difference (expressed in mean solar time) in the lengths of the days at these two times. Assume the latitude of the

place to be 30° N., and the greatest declination of the sun 23° 27′. Disregard corrections for refraction and parallax.

Ans. Difference of azimuth = 54° 42′.
Difference of lengths of days = 3 hrs. 52 min.

10. In latitude 30° 18′ S., longitude 123° 40′ E., the following sun observation was taken at 4 hrs. 45 min. p.m. :—

Alt., . .	22° 28′ 30″, 258° 43′ 30″	$\lfloor\odot$	R.M.
Co-alt., . .	67° 55′ 30″, 258° 51′ 30″	$\overline{\odot}\rfloor$	357° 46′.

The sun's declination for the day, G.M.N., was 20° 19′ 02″ S., and for the preceding day 20° 06′ 16″ S., the semi-diameter being 16′ 14″. Find the true bearing of the R.M.

Ans. 328° 08′ 08″.

11. Find the bearing and altitude of a star at its Eastern elongation, also the mean time of elongation. The latitude of the place is 31° S., the longitude 8 hours West, the R.A. of star is 6 hrs. 21 min. 30 sec., its declination 52° 37′ S., and sidereal time at G.M.N. on the day of observation 14 hrs. 28 min.

Ans. Bearing, 134° 54′
Altitude, 40° 25′.
Mean time, 11 hrs. 38 min. 55 sec.

12. In latitude 25° 58′ N. Polaris was observed at its Eastern elongation, its declination for the date being 88° 44′ 20″. Compute the azimuth of the star.

Ans. 1° 24′ 10″.

13. At Adelaide (latitude 34° 55′ 38″ S., longitude 9 hrs. 14 min. 20 sec. E.) a forenoon observation was made of the sun on June 24th, 1914.

From two observations taken with F.R. the mean angle between R.M. and sun was 85° 34′ 05″, the mean altitude 24° 03′ 50″. The mean time was 10 hrs. 7 min. 30 sec. a.m. (standard time of meridian 9 hrs. 30 min. E.). With F.L. the mean angle between R.M. and sun was 87° 21′ 00″, the mean altitude 24° 55′ 07″, the mean standard time 10 hrs. 15 min. 30 sec. a.m. The sun's declination at G.M.N. on June 23rd was 23° 26′ 51·9″ N., the variation in one hour being 1·26″ on the 23rd, and 2·29″ at noon on the 24th. The angle between the sun and R.M. was measured from the sun to the right. Determine the true bearing of the R.M. Allow for refraction and parallax.

Ans. 118° 09′ 28″.

14. During the evening of the date 28th July, 1914, several bearings of α Centauri were observed when it was near elongation. Find the true bearing of the referring lamp, which was assumed to be 179° 00′ 00″.

OBSERVATIONS.

Statute Time, 10 Hours East of Greenwich.	Bearing of α Centauri.	Weight of Observation.
10 hrs. 32 min. 15 sec.	217° 29′ 10″	2
10 hrs. 37 min. 20 sec.	217° 32′ 44″	3
10 hrs. 42 min. 06 sec.	217° 34′ 48″	3
10 hrs. 47 min. 12 sec.	217° 36′ 02″	2
10 hrs. 52 min. 05 sec.	217° 35′ 25″	4

Longitude 9 hrs. 39 min. 54 sec. E., Latitude 37° 49′ 53″ S.
R.A. of star 14 hrs. 33 min. 48 sec., Declination 60° 29′ 16″ S.
Sidereal time, G.M.N., July 28th, 1914, 8 hrs. 21 min. 13·93 sec.

CHAPTER IX.

THE DETERMINATION OF LATITUDE.

THERE are many possible ways by which the surveyor may determine the latitude of the place of observation, but, as in the previous chapter, we shall here confine our attention to the most practicable and most generally used methods.

First Method—By Meridian Altitudes of Sun or Star.—This is a very convenient and simple way of finding latitude, where the greatest possible precision is not required, and depends upon the fact we have already discussed in Chapter III. that the altitude of the celestial pole is equal to the latitude of the place of observation. It follows that the latitude may be at once obtained by observing the meridian altitude of a body whose declination or polar distance is known. This is the method commonly used by the sailor at sea, the altitude of the sun at apparent noon being observed with a sextant. In Fig. 37, if O denote the position of the observer, Z the zenith point, P the celestial pole, then if an object be observed at S_1, we have $A\,P = A\,S_1 - P\,S_1$, or latitude = meridian altitude − polar distance. This might represent the position of a circumpolar star at its upper culmination. If it were observed at lower culmination it would be in the position S_2, and in that case $AP = A\,S_2 + P\,S_2$, or latitude = meridian altitude + polar distance.

In other cases the object observed may be on the opposite side of the zenith to P. If E denotes the point where the celestial equator intersects the meridian, the body may be at S_3 or S_4. Since $B\,E + P\,A = 90°$, it follows that B E = the co-latitude. Then at S_3 we have B E = B S_3 − E S_3, or co-latitude = meridian altitude − declination. When the body is in the position S_4 its declination will be South if the observation is made in northern latitudes or north if the place is in South latitude. In that case we have B E = B S_4 + E S_4, or co-latitude = meridian altitude + declination.

Thus in all cases the latitude can be very simply obtained provided that we know the declination of the celestial body.

The observed altitude must be corrected for refraction as discussed in Chapter VII., and as the amount of this correction depends upon the pressure and temperature of the air, it is necessary, if the correction is to be made as accurately as possible, that thermometer and baro-

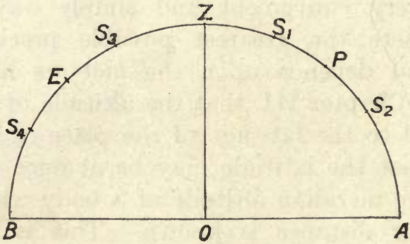


Fig. 37

meter readings should be taken at the time of observation. Usually it will be sufficient to take the refraction correction straight from the table of mean refractions, without troubling to allow for the difference between the actual temperature and pressure from that for which the table of mean refractions is made out, because the maximum change in the refraction due to an alteration of temperature only amounts to about 3″ per 10° F., and for a change of pressure to about 5″ per inch of barometer.

In the case of the sun, since it is the altitude of the upper or lower limbs that must be observed, and it is the altitude of the sun's centre that is required, a correction must be made for its semi-diameter. Another correction also must be made to allow for parallax. Both of these are found from the Nautical Almanac. With observations upon the fixed stars neither of these corrections is needed.

If the altitude of the sun is observed with a sextant on land an artificial horizon must be used, in which case the double altitude is measured. The following is an illustration of such an observation, made in South latitude :—

Double altitude sun's lower limb .	64° 13′ 10″
Index error + . . .	4′ 5″
	2) 64° 17′ 15″
	32° 08′ 37·5″
Refraction —	1′ 55″
	32° 06′ 42·5″
Parallax + ...	7″
	32° 06′ 49·5″
Semi-diameter + . . .	15′ 50″
Altitude of sun's centre, . . .	32° 22′ 39·5″
Declination N, ...	19° 47′ 53″
Co-latitude, ...	52° 10′ 32·5″
∴ latitude = ...	37° 49′ 27·5″

With observations upon the sun, if the local mean time is known, the time of apparent noon may be found by applying the equation of time as found from the Nautical Almanac. The altitude of the sun's lower or upper limb may then be observed at the proper instant as measured by the watch. The effect of an error in time will depend upon the latitude of the observer and the declination of the sun. In latitude 45°, with the sun on the celestial equator, an error of 1 minute in time will produce an error of only 2 seconds in the measured altitude. Under the same conditions if the time is wrong by as much as 10 minutes, the altitude measured will be too small by 49 seconds. So that for the ordinary purposes of the surveyor, when the observation is made in this way, it is not necessary to know the local time with great exactness. If the approximately correct time is not known the sun is followed by the observer and the altitude measured when it attains its greatest value.

With observations upon the stars the same general principles will apply. With close circumpolar stars it is possible to take two observations for altitude upon the same star, the face of the instrument being reversed after the first reading is taken. When this can be done the accuracy of the determination is increased, but as a rule the altitude changes too rapidly for this to be possible. In latitude 30°, for instance, the altitude of a star having a polar distance of 30° is 43″ less 5 minutes before and after its culmination than when on the meridian.

Zenith Pair Observation of Stars.—A great improvement upon the accuracy of simple meridian observations may be effected by making observations upon two stars which culminate at approximately equal altitudes on opposite sides of the observer's zenith. The altitude of one star having been observed at culmination, the face of the instrument is reversed and the meridian altitude of the second star is then measured. The two stars must, of course, be chosen so that the second culminates at a convenient interval after the first. The method is commonly referred to as that of latitude determination by "zenith pair observations." No attempt is made to take two observations on the one star, and the combination of the two results largely eliminates errors of refraction and errors due to the graduation of the vertical arc. Thus, in Fig. 37, if S_1 and S_3 denote the two observed stars, we obtain from the observation upon S_1

$$\text{lat.} = A\,S_1 - P\,S_1 = A\,S_1 - p_1, \qquad (1)$$

and from the observations upon S_3

$$\text{co-lat.} = B\,S_3 - E\,S_3 = B\,S_3 - (90^\circ - p_3)$$

$$\therefore \qquad \text{lat.} = 180^\circ - B\,S_3 - p_3 \qquad (2)$$

Taking the average of the determinations (1) and (2), we obtain

$$\text{lat.} = 90^\circ + \frac{A\,S_1 - B\,S_3}{2} - \frac{p_3 + p_1}{2}.$$

Thus, in the final determination it is the *difference* of the measured altitudes A S_1 and B S_3 that is required, and as any error in the allowance made for refraction will affect both the altitudes alike, the error will practically disappear when we subtract them. Consequently, the method almost eliminates errors due to an uncertain knowledge of the refraction, and also enables instrumental errors to be largely eliminated by taking two separate observations with opposite faces of the instrument.

If the local time and consequently the sidereal time is known with fair accuracy, the best way is to intersect each star at the instant when the sideral time is equal to the star's right ascension. This is found from the Nautical Almanac, and the two stars will be selected for convenience, if possible, so that their right ascensions differ by from 10 to 30 minutes. If the time is not known accurately, then the telescope must be directed to the true meridian, and the altitude measured when the star intersects the vertical wire. Readings must be taken also of the barometer, thermometer, and alidade level.

The following example is taken from the Western Australian Handbook for Surveyors :—

Date, 1st May, 1910.
θ Argus — observed altitude (South) = 58° 01′ 30″.
Alidade level — O = 5·8, E = 3·2.
1 Division of level = 15″.

Barometer = 30·52″. External thermometer = 72·5°. Attached thermometer = 71°.

Note.—O means object end of telescope. E means eye end of telescope.

Compute refraction from "Bessel's Refractions," as given on pp. 430, 431 Chambers' Log Book, from the formula :—

True ref. = mean ref. × B × t × T.

Mean refraction alt. 58° 01′	= 36·1″,	. .	1·55751
B for 30·52″	= 1·032″,	.	0·01368
t for 71° F.	= 0·997″,	.	9·99870
T for 72·5° F.	= 0·955″,	.	9·98000
True refraction	= 35·5″,	. .	1·54989

Obsd. alt.	= 58° 01′ 30″		
Ref.	= 35·5″		Level.
	58° 00′ 54·5″	O	5·8
Level +	= 19·5″	E	3·2
			2) 2·6
True alt.	= 58° 01′ 14″		1·3 × 15″ = 19·5″
Polar distance	= 26° 04′ 19·3″		
Latitude	= 31° 56′ 54·7″		

Date, 1st May, 1910.

l Leonis — Obsd. Z.D. (North) = 42° 57′ 00″.

Alidade Level, O = 3·5, E = 5·5.

Barometer = 30·55″, External Thermometer = 72·0°.

Attached Thermometer = 71·0°.

			Logs.
Mean ref. alt. 47° 3′	= 53·7″,	. . .	1·72997
B for 30·55″	= 1·032,	. . .	0·01368
t for 72·0°	= 0·997,″	. . .	9·99870
T for 71·0°	= 0·956″,	. . .	9·98046
True refraction	= 52·8″,	. . .	1·72281

			Level.
Obsd. alt.	= 47° 03′ 00″	E	5·5
Ref.	= 52·8″	O	3·5
			2) 2·0
	47° 02′ 07·2″		1·0 × 15″ = 15″
Level	= 15·0″		
True alt.	= 47° 01′ 52·2″		
Declination	= 11° 01′ 16·1″		
Co-latitude	= 58° 03′ 08·3″		
Latitude	= 31° 56′ 51·7″		

Deduced latitude—θ Argus (South),	. . .	31° 56′ 54·7″
l Leonis (North),	. . .	31° 56′ 51·7″
	Mean =	31° 56′ 53·2″

Meridian Altitudes of a Star at both Lower and Upper Culminations. — If the meridian altitudes of a star be observed at both lower and upper culminations, then, if these be separately corrected for refraction, the mean of the two altitudes will give the altitude of the celestial pole, which is equal to the latitude of the place. The method does not require a knowledge of the declination of the star, but as this information is always to be obtained in the Nautical Almanac, there is no practical advantage to the surveyor, save perhaps in very exceptional cases. On the other hand, the long interval necessary between the two observations is a very practical inconvenience. Consequently, the method is not one in practical use amongst surveyors, although it is employed by astronomers at fixed observations.

Second Method—By Circum-Meridian Observations.—Observations of stars or the sun taken near to the meridian are commonly spoken of as circum-meridian observations. By taking a series of altitudes of a star or the sun for some few minutes both before and after it crosses the meridian, instrumental errors may be largely eliminated, and by proper methods of reduction the results may be used to give a very accurate determination of latitude. It is necessary to have the means of accurately noting the time of each observation, and then each altitude may be corrected or reduced so as to give us the corresponding altitude on the meridian itself. Thus a series of "circum-meridian" altitudes becomes equivalent to a series of measurements taken on the meridian itself, and in the taking of such a set of observations the instrument may be reversed and its errors eliminated in a way that is not possible with a single meridian observation. Still greater precision may be attained by taking such observations upon equal numbers of stars North and South of the Zenith, at approximately equal altitudes.

In Fig. 38, let Z be the Zenith, P the celestial pole, and S the observed star. As this is to be near the meridian, the angle S P Z will be small.

Let $z =$ S Z, the Zenith distance,
$p =$ S P, the polar distance of the star,
$c =$ P Z, the co-latitude,
$t =$ the hour angle S P Z.

Then, from the triangle S P Z,

$$\cos z = \cos c \cos p + \sin c \sin p \cos t. \qquad (1)$$

Let x be the correction that has to be applied to the observed zenith distance, z, in order to deduce the zenith distance when the star is on the meridian.

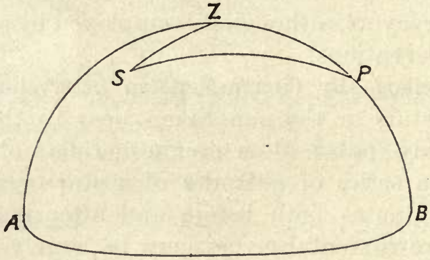


Fig. 38.

Then meridian zenith distance $= z - x = p - c$.

If, now, in equation (1) we write $\cos t = 1 - 2 \sin^2 \frac{t}{2}$, we get

$$\cos z = \cos (c - p) - 2 \sin c \sin p \sin^2 \frac{t}{2}.$$

$$\therefore \quad \cos z - \cos (z - x) = - 2 \sin c \sin p \sin^2 \frac{t}{2}.$$

$$\therefore \quad 2 \sin \frac{x}{2} \sin \left(z - \frac{x}{2}\right) = 2 \sin c \sin p \sin^2 \frac{t}{2}.$$

$$\therefore \quad \sin \frac{x}{2} = \frac{\sin c \sin p \sin^2 \frac{t}{2}}{\sin \left(z - \frac{x}{2}\right)}.$$

If x is small, we may now replace $\sin \frac{x}{2}$ by the circular measure of $\frac{1}{2}x$, which is $\frac{1}{2}x \sin 1''$, provided that x is measured in seconds of arc. Also, we shall make very little difference to the result if, instead of the $\sin\left(z - \frac{x}{2}\right)$ of the denominator we write $\sin(z-x) = \sin(p-c)$. Thus, if x is the correction to be applied in seconds of arc, we obtain

$$x = \frac{\sin c \sin p}{\sin (p-c)} \frac{2 \sin^2 \frac{t}{2}}{\sin 1''},$$

or, in the form in which it is more usually written, if a denotes the observed altitude, A the altitude on the meridian, l the latitude, and n the declination,

$$A - a = \frac{\cos l \cos n}{\cos A} \cdot \frac{2 \sin^2 \frac{t}{2}}{\sin 1''}.$$

It will be noticed that if a series of observations are taken upon the same star, the first factor in this expression, *i.e.* $\dfrac{\cos l \cos n}{\cos A}$, is the same for them all. We will denote this by B. If we write $m = \dfrac{2 \sin^2 \frac{t}{2}}{\sin 1''}$, we have

$$A = a + B\, m.$$

The value of m in seconds may be computed, knowing the value of t, or more conveniently it may be taken from tables such as are given in Chauvenet's Astronomy or from the abbreviated table given at the end of this chapter.

Thus, if a_1, a_2, a_3, etc., denote a series of observed circum-meridian altitudes of the same star, and m_1, m_2,

m_3, etc., are the corresponding values of m, we obtain a series of values for the corresponding meridian altitudes given by the equations

$$A_1 = a_1 + B\, m_1$$
$$A_2 = a_2 + B\, m_2$$
$$A_3 = a_3 + B\, m_3, \text{ etc.}$$

Therefore, if we denote by A_0 the mean of the deduced meridian altitudes, by a_0 the mean of the actual observed altitudes, and by m_0 the mean of the computed factors m, we have

$$A_0 = a_0 + B\, m_0.$$

With the aid of tables for m, the reduction of the observations thus becomes extremely simple. We take the mean of the values of m, multiply by B, and add the product to the mean of the observed altitudes.

The deduced mean meridian altitude is then corrected for refraction and the latitude is computed as an ordinary meridian altitude observation.

The value of B involves both the latitude and the meridian altitude, since

$$B = \frac{\cos l \cos n}{\cos A.},$$

but the value of l used in this is the approximate latitude as deduced either from the map or from a simple meridian observation. The value of A used is the meridian altitude computed from the approximate latitude and the known declination of the star. The approximate value of B thus deduced is quite sufficiently accurate, when multiplied by m, to give the correction required. A still higher degree of accuracy may be attained by repeating the calculation, using for B the value of the latitude as first computed.

Before starting the actual observations, it is necessary to calculate the time of the star's meridian transit. The observations should then be made within about ten minutes on each side of this. The t in the formula is the interval of sidereal time between the instant of actual observation and the instant of meridian transit, expressed in angular measure at the rate of 15° per hour.

The method involves an accurate knowledge of the local time, and is then capable of a high degree of precision. To get the best results the errors should be balanced by taking an equal number of observations on stars both North and South of the Zenith. An equal number should be selected on each side at approximately equal altitudes. The errors are likely to be greatest for stars observed near to the Zenith, especially when the place of observation is near to the equator. The range of observed altitudes should, if possible, lie between 40° and 75° above the horizon, and the closer the stars are observed to the meridian the better will be the results.

More Exact Methods of Reduction of Circum-Meridian Observations.—The approximate formula that we have given is the one usually adopted for the reduction of circum-meridian observations. A still closer approximation may be obtained by using the more elaborate formula

$$A = a + B\,m + C\,m',$$

where $C = B^2 \tan A$ and $m' = \dfrac{2 \sin^4 \frac{1}{2} t}{\sin 1''}$,

A and B having the same significance as before.

The correction introduced by the third term in the formula is usually very small when the observations are made close to the meridian. If the value of t in minutes does not exceed two-fifths of the Zenith distance of the star in degrees, then it can be shown that the correction introduced by the term $C\,m'$ is never more than 1″, so that the more exact formula is only required where the highest precision possible is sought.

This may be obtained in a manner similar to that employed in Chap. VIII. for the corresponding formula for circum-elongation observations for azimuth.

The Limits of Time for the Observations.—According to what we have just seen, the greatest interval of time in minutes between any observation and the instant of meridian transit should not exceed two-fifths of the zenith distance of the star in degrees if the error in reducing the observation to the meridian is to be limited to 1″. It is not possible to work so precisely as this with the instruments commonly used, and the time may be extended somewhat beyond this limit. In general, it seems a good rule to say that the greatest value of t in minutes of time should not exceed one-half of the zenith distance in degrees. Thus, if the altitude of the star is 50°, the observations may be made within 20 minutes on each side of the meridian transit. In that particular case the maximum error would still only amount to 1″, but in other cases the error may be somewhat greater if this rule is followed, but never so much as 3″, provided that the star is not within 10° of the zenith.

Circum-Meridian Observations of the Sun.—As a general rule, it is more convenient for the surveyor to make observations upon the sun than upon the stars, and exactly the same method as we have described may be followed for circum-meridian observations of the sun. Obviously the sources of error cannot be balanced in the same way as with stars by taking observations both North and South of the zenith, so that such precise work is not possible. There is another difficulty arising from the fact that the sun's declination is not constant and, if the observations extend over 30 minutes, it may vary by as much as 30″. If, however, a similar number of observations are made both before and after apparent noon, the errors will very nearly balance in the mean, provided that in the computations the value of the declination used is the value at apparent noon. This is not exact, but sufficiently so for all but the most precise work.

An even number of observations should be made, usually eight.

The first observation will be to the sun's upper limb with F.R. Then two in succession to the lower limb with F.L., next two in succession to the upper limb, the instrument being reversed, once more with F.R. Two more to the lower limb with F.L., and finally one to the upper limb with F.R. With this order the sun's diameter is eliminated in the mean. The alidade level should be read and recorded at each observation. The method of recording and the calculation is shown in the accompanying example :—

EXAMPLE OF CIRCUM-MERIDIAN OBSERVATION OF SUN FOR LATITUDE.

Place, ... Survey Office, Adelaide.
Longitude, ... 9 hrs. 14 min. 20·3 sec.
Date, ... July 4th, 1914.

Sun's Limb Observed.	Face of Instrument.	Standard Time.	Vertical Circle.		
			A.	B.	Mean.
		H. M. S.			
U	R	12 12 58	32° 21′ 45″	32° 21′ 30″	32° 21′ 37″
L	L	12 14 53	31° 51′ 30″	31° 51′ 30″	31° 51′ 30″
L	L	12 16 57	31° 52′ 00″	31° 52′ 00″	31° 52′ 00″
U	R	12 19 00	32° 23′ 00″	32° 23′ 00″	32° 23′ 00″
U	R	12 20 00	32° 23′ 00″	32° 23′ 00″	32° 23′ 00″
L	L	12 21 56	31° 52′ 00″	31° 52′ 00″	31° 52′ 00″
L	L	12 23 58	31° 51′ 50″	31° 51′ 50″	31° 51′ 50″
U	R	12 25 56	32° 21′ 50″	32° 21′ 50″	32° 21′ 50″

Mean observed altitude = 32° 07′ 05·9″.

Preliminary Computations.

For Approximate Latitude.

Obsd. Max. Alt. of Sun, . .	32° 23′ 00″
R and P, . .	1′ 23″
	32° 21′ 37″
— Semi-diam., .	15′ 45·4″
Corrected Altitude,	32° 05′ 51·6″
Declination, .	22° 58′ 23·1
Co-latitude, .	55° 04′ 14·7″
Latitude, . .	34° 55′ 45·3″

For Time of Apparent Noon.

	H.	M.	S.
Equation of Time, on July 4th,	0	4	2·47
Correction for Long., . .	0	0	4·24
Corrected Eq. of Time, .	0	3	58·23
L.M.T. of Apparent Noon, .	12	03	58·23
Diff. for Standard Meridian, .	0	15	39·7
Standard Time of App. Noon,	12	19	37·9

For B.

log Cos Latitude, .	9·913739
log Cos Declination,	9·964112
log Sec Altitude, .	10·072042
log B, . . .	9·949893
B =	0·891

COMPUTATION FOR LATITUDE.

t (Mins.)	t (Secs.)	m
6	40	87·3″
4	45	44·3″
2	41	14·1″
	38	0·8″
	22	0·3″
2	18	10·4″
4	20	36·9″
6	18	77·9″
	8″)	272·0″

m_0, ...	34·0″
m_0 B, ...	30·3″
Mean observed altitude, .	32° 07′ 05·9″
A_0, ...	32° 07′ 36·2″
Refraction − . .	1′ 31″
	32° 06′ 05·2″
Parallax + . . .	07″
Corrected altitude, .	32° 06′ 12·2″
Declination, . . .	22° 58′ 23″
Co-latitude, . . .	55° 04′ 35·2″
Latitude, . . .	34° 55′ 25″

Third Method — Latitude by Prime Vertical Transits.— The Prime Vertical has been already defined as the vertical plane at right angles to the meridian, running truly East and West. Stars with polar distances less than 90° and greater than the distance of the pole from the zenith—*i.e.*, greater than the co-latitude of the place—will cross the prime vertical twice in a sidereal day. If the interval of time between the East and West transits of a star be measured, and the declination of the star be known, then the latitude can be readily computed. Thus, in Fig. 39, let E Z W represent the prime vertical of the observer, Z being the zenith. Let P be the celestial pole and A C B the portion of the star's path described on the same side of the prime vertical as the pole. A and B are the points where the star's path intersects the prime vertical. If A and B respectively joined to the pole P by arcs of great circles, P A and P B will each be equal to the star's polar distance or to the complement of its declination. Then, in the spherical triangle A Z P, the angle at Z is a right angle, P Z = the co-latitude, P A = the star's polar distance, and the angle A P Z, if turned into time at the rate of 15° per hour, will represent half the interval between the transits at A and B measured in sidereal time.

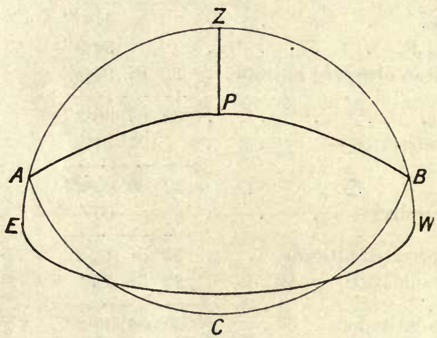


Fig. 39.

From Napier's Rules we have

$$\cos A P Z = \tan P Z \times \cot A P,$$

whence $$\tan \text{latitude} = \tan \text{declination} \times \sec t,$$

where t = half the interval of sidereal time between the transits expressed in angular measure.

By this method the errors due to uncertainty with regard to refraction are largely eliminated, because the times of transit are observed instead of altitudes. The method does not require a knowledge of the exact local time, as it is an interval of time that has to be measured, consequently it is sufficient for the surveyor to have a watch or clock whose rate is known.

It will be obvious that in places where the elevation of the celestial pole is small—that is to say, in places near the equator—the paths of such stars as move across the prime vertical will intersect it very obliquely, and it will not be possible to secure a good determination of the exact time of intersection. A precise measurement will be more easy in places of higher latitude.

The Effect upon the Determination of an Error in the Measurement of the Time Interval.—To make a determination of latitude by the method just described, the surveyor has to set out the direction of the prime vertical, and also to measure the time interval between the East and West transits. In order to judge therefore of the degree of precision of which the method is capable we require to investigate the effect of small errors in each of these measurements.

If we denote the latitude by l and the declination of the observed star by d, we have

$$\tan l = \tan d \,.\, \sec t. \quad . \quad . \quad . \quad (1)$$

If a small error y is made in the measurement of t, and x is the corresponding error made in the latitude,

$$\tan (l + x) = \tan d \,.\, \sec (t + y).$$

Expanding and writing $\tan x = x$, $\cos y = 1$, $\sin y = y$ since x and y are small, we have

$$\frac{x + \tan l}{1 - x \tan l} = \frac{\tan d}{\cos t - y \sin t}.$$

$\therefore$ neglecting the product of the small quantities $x\,y$, we get

$$x \cos t + \tan l \cos t - y \tan l \sin t = \tan d - x \tan d \tan l.$$

Making use of (1), this becomes

$$x\left(\frac{\tan d}{\tan l} + \tan d \tan l\right) = y \tan l \sqrt{\left(1 - \frac{\tan^2 d}{\tan^2 l}\right)},$$

$$\therefore \qquad x\,\frac{\tan d \sec^2 l}{\tan l} = y \tan l \sqrt{\left(1 - \frac{\tan^2 d}{\tan^2 l}\right)},$$

$$\therefore \qquad x = y\,\frac{\sin 2\,l}{2} \sqrt{\left(\frac{\tan^2 l}{\tan^2 d} - 1\right)}.$$

The student who understands differential calculus can obtain this result at once by differentiating equation (1), keeping d constant.

From this equation we get the important practical deductions that if d is nearly $= l$, x will be very small, and that if d is nearly $= 0$, x will be very large. So that it would seem that the stars most suitable for observation are those whose declinations are nearly equal to the latitude. A star having a declination the same as the latitude would pass through the zenith point, and the declination must be somewhat less than the latitude for the method to be possible. On the other hand, a star with zero declination would pass through the E. and W. points on the horizon at the prime vertical for all latitudes, the interval of time between its transits would be exactly six hours no matter what the position of the observer, and no determination of latitude could be made. It would apparently follow, then, that the best stars to select are those that cross the prime vertical near the zenith. But a star crossing the prime vertical very near to the zenith intersects it so obliquely that it is not possible to make an accurate determination of the time of transit. The distance from the zenith, at which the path of the star will make a sufficiently large angle with the prime vertical to enable a good measurement of the transit to be made, will depend upon the latitude of the observer. And the practical conclusion is that the stars observed should be as high up on the prime vertical as is consistent with an exact determination of the time of transit. Stars which cross it low down must be avoided, as they lie near the celestial equator, and the error in latitude produced by a slight error in time is then very large.

A definite calculation will give a better idea of the effect of a defective measurement of the time interval. If we take a place in latitude 30°, and suppose the observation to be made on a star with a declination of 10°, then $x = 1{\cdot}3\,y$. Now t in our formula is half the total time interval between the transits, so if this whole interval is in error to the extent of one second of time, $y =$ half a second. But half a second of time is equivalent to 7·5 seconds of arc, and this multiplied by 1·3 gives 9·7 seconds of arc as the error in latitude caused by an error of one second in the time interval.

If in the same latitude the star observed has a declination of 20°, then, from the same formula, $x = {\cdot}52\,y$. In this case a mistake of 1 second in the total time interval will cause an error of 3·9 seconds in the latitude. If the declination is 25°, $x = {\cdot}32\,y$, and the corresponding error in latitude is 2·4 seconds. In higher latitudes the errors are still greater.

Clearly, even if the surveyor is to be content with a determination of latitude to the nearest minute of arc, he must be able to rely upon his measurement of the time interval within a few seconds.

The Effect of an Error in the Direction of the Prime Vertical.—The error arising from a defective setting out of the prime vertical is not nearly so serious, because, if this is marked out so that the time of the Eastern transit of the star is earlier than it should be, then the time of the Western transit will be correspondingly hastened, so that the interval between the transits will be very little different to that when the prime vertical is correctly located. Thus, in latitude 30°, the measurements being made on a star with a declination of 20°, even if the prime vertical is set out as much as 1° out of its true position, the resulting error in the latitude determination is less than 1 minute of arc. So that a comparatively rough determination of the prime vertical is sufficient for the surveyor's purpose. It is, of course, most important that the instrument shall be in accurate adjustment, so that it will sweep out a truly vertical circle. But instrumental errors may be largely eliminated by taking observations on alternate nights with the instrument reversed.

Although the method is capable of giving results of great precision, the practical inconvenience caused by the long interval between transits and the necessity for exact time measurements rather put it out of court as a suitable method for ordinary surveyors in the field.

The same method may be applied, with some modification of formulæ, to any vertical circle whatever. But the prime vertical circle is the most suitable for accurate work.

Striding Level Correction to Prime Vertical Observations.—The striding level should always be used with prime

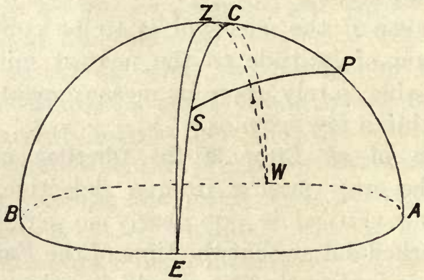


Fig. 39*a*.

vertical observations as the resulting determination of the latitude is in error by an amount equal to the angle which the transverse axis of the telescope makes with the horizontal. Thus, in Fig. 39*a*, if P denotes the celestial pole, Z the zenith, and E W the East and West points on the horizon, then, if the striding level shows an error in the horizontality of the transverse axis of the telescope, the circle upon which the observations are actually made will be E C W instead of the true prime vertical E Z W. The star is observed to transit at the point S, the angle S P C $= t$, and the angle S C P is a right angle.

Thus, we shall get

$$\cot CP = \tan \text{declination} \times \sec t.$$

The true co-latitude is then $CP \pm ZC$, the $+$ sign being taken if, as in the figure, C is on the same side of Z as P, and the $-$ sign being used if C and P are on opposite sides of Z. This is determined by the direction of the level error, and $ZC =$ the angular measure of the level error.

Thus, to make the correction, the computation for latitude is made in the ordinary way, and then we add or subtract the striding level error.

Example.—*At a place in S. latitude the interval between the passage of Sirius across the prime vertical is* 6 *hrs.* 09 *min.* 19·1/3 *sec. mean time. The mean readings of the bubble on striding level were* 10 *N. and* 14 *S., each division being* = 20″. *The declination of the star is* 16° 35′ 33″ *S. Determine the latitude.*

6 hrs. 09 min. 19·1/3 sec. of mean time
= 6 hrs. 10 min. 20 sec. of sidereal time
= 92° 35′ 00″ of arc

tan lat. = tan dec. × sec. 46° 17′ 30″.

tan dec., ...	9·4741732
cos 46° 17′ 30″, ...	9·8394702
	9·6347030

∴ lat. = 23° 19′ 37″.

But the striding level error necessitates a correction

$$= \frac{14 - 10}{2} \times 20 = 40''.$$

As the South end of the transverse axis is the higher, the derived latitude is too small.

∴ corrected latitude = 23° 20′ 17″ S.

Fourth Method—By the Altitude of the Pole Star at any Time.—Provided that the exact local time and the approximate longitude are known, the latitude may be found from an altitude observation of a close circumpolar star at any time. In the Northern Hemisphere the Pole Star is commonly selected for this purpose, and special tables are given in the Nautical Almanac for reducing the observations. In the Southern Hemisphere unfortunately there is no bright star sufficiently near to the Pole to make the method a convenient one for the surveyor.

In Fig. 40, let S be the circumpolar star, Z the zenith, and P the pole as before. Then, with the previous notation, if

$z =$ S Z, the zenith distance,
$p =$ S P, the polar distance of the star,
$c =$ P Z, the co-latitude,
$t =$ the hour angle S P Z.

From the triangle S P Z we have

$$\cos z = \cos c \cos p + \sin c \sin p \cos t,$$

or, if a is the observed altitude, and l the latitude

$$\sin a = \sin l \cos p + \cos l \sin p \cos t.$$

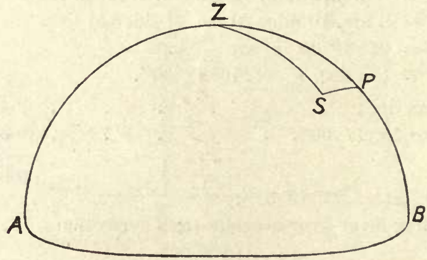


Fig. 40.

Now a will differ from l by a small quantity, which is always less than p. In the case of the Pole star p is also small, being about 1° 10′ at present. Let

$$a = l + x,$$

where x is a small correction.

$$\therefore \sin l \cos x + \cos l \sin x = \sin l \cos p + \cos l \sin p \cos t.$$

$$\therefore \sin l\,(1 - \frac{x^2}{2} + \ldots) + \cos l\,(x - \frac{x^3}{6} + \ldots)$$

$$= \sin l\,(1 - \frac{p^2}{2} + \ldots) + \cos l \cos t\,(p - \frac{p^3}{6} + \ldots).$$

Neglecting the square and higher powers of x and p in this equation, we get $x = p \cos t$, which is the value of x to a first approximation.

Next, retaining the squares of x and p, but neglecting the higher powers, we get

$$x \cos l = p \cos l \cos t - \frac{p^2}{2} \sin l + \frac{x^2}{2} \sin l.$$

Substituting for x^2 the value $p^2 \cos^2 t$, we obtain then as a second approximation

$$x = p \cos t - \tfrac{1}{2} \tan l \sin^2 t \,.\, p^2.$$

The second term in this expression is very small, and as $\tan l$ differs from $\tan a$ by only a small quantity, the difference when multiplied by p^2 will be too small to take into account, so that we may write

$$x = p \cos t - \tfrac{1}{2} \tan a \sin^2 t \; p^2.$$

In this formula x and p are in circular measure, but if x and p are measured in seconds we may write

$$x = p \cos t - \tfrac{1}{2} p^2 \tan a \sin^2 t \sin 1'',$$

so that we have for the latitude

$$l = a - p \cos t + \tfrac{1}{2} p^2 \tan a \sin^2 t \sin 1''.$$

The formula is, of course, an approximation only, but it can be shown that it is sufficiently accurate to give the result within 1″ of the truth.

To determine t, the sidereal time must be known accurately at the moment of observation, and t is then the difference between the sidereal time and the right ascension of the star turned into angular measure.

Four altitudes should be taken in as quick succession as possible, one with F.R., two with F.L., and then again one with F.R., the alidade level being read at each observation, and the chronometer times noted. The mean of the altitudes and the mean of the chronometer times are then taken as the basis for the reduction as a single observation.

A Rough Method for the Determination of Latitude by Noting the Rate at which Altitude of Sun or Star Changes near the Prime Vertical.—This is only a very rough and approximate method at best, but it is interesting because of its simplicity, and because it requires no knowledge of either the local time or the declination of the body observed. But it is not to be classed along with the previous methods.

In Fig. 41, let Z be the zenith point, P the celestial

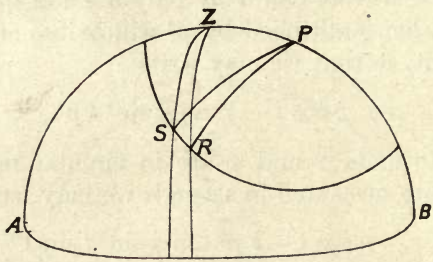


Fig. 41.

pole, and R S two consecutive positions of the sun or star. The change of altitude will be measured by the difference between the arcs Z R and Z S, and the interval of time between the two positions will be measured in angular measure by the angle S P R.

In the triangle Z P R, Z R = zenith distance = z, P Z = co-latitude = c, P R = polar distance = p, R Z P = azimuth measured from elevated pole = A, Z P R = hour angle = B.

In the triangle Z P S, suppose that z has become changed to $z - y$, and B to B $- x$, c and p remaining unaltered.

Then from the formulæ of spherical triangles, we have $\cos z = \cos p \,.\, \cos c + \sin p \,.\, \sin c \,.\, \cos B$, and $\cos (z-y) = \cos p \cos c + \sin p \sin c \cos (B-x)$.

Subtracting these expressions, and regarding x and y as small quantities, so that we may write $\cos x = 1$, $\sin x = x$, etc., we obtain

$$y \sin z = \sin p \sin c \sin B \,.\, x.$$

But
$$\frac{\sin z}{\sin B} = \frac{\sin p}{\sin A},$$

$\therefore$
$$y = x \,.\, \sin c \sin A,$$

or
$$\cos \,.\, \text{latitude} = \frac{y}{x} \text{cosec} \,.\, \text{azimuth}.$$

Thus, in order to determine the latitude, all we have to do is to measure the change of altitude y that takes place in a given time whose angular measure is x. If $t =$ the interval of time in seconds, $x = 15\,t$ seconds of arc.

As the ratio $\frac{y}{x}$ is to be multiplied by cosec A, and the observation is made near the prime vertical, an error in the azimuth A will have but a small effect upon the result.

A convenient way of making the observation is to take the time required by the sun, in the afternoon or early morning, to cross the horizontal wire of the telescope, observing at the same time the sun's approximate bearing. For an afternoon observation, bring the sun's lower limb into contact with the wire and start the stop watch. When the sun is about bisected by the wire, read the approximate azimuth of its centre. Stop the watch at the instant that the upper limb becomes tangent to the wire.

Example.—At a place in South Latitude on March 17th, the sun took 2 min. 46·4 sec. to transit the horizontal wire of a theodolite, the bearing of its centre being 289° 20′.

$$\text{Diameter of sun} = 32'\,11{\cdot}3'' = 1{,}931{\cdot}3''$$

$$15 \times 2'\,46{\cdot}4'' = 15 \times 166{\cdot}4 = 2{,}496$$

$$\cos \text{lat.} = \frac{1{,}931{\cdot}3}{2{,}496} \times \operatorname{cosec} 109°\,20'.$$

1,931·3, ...	3·2858497
cosec 109° 20′ = sec 19° 20′, . .	10·0252082
	13·3110579
2,496, ...	3·3972446
cos 34° 55′, ...	9·9138133

Therefore, the latitude is 34° 55′ S.

It will be found on trial in this example that if the azimuth is 1° out, the computed latitude is about 30′ in error, and we must know the azimuth of the sun within 2′ if we wish to find the latitude to the nearest minute. If the observation had been made with the sun nearer to the prime vertical, however, an error in azimuth would not produce anything like so serious an effect.

To get anything like accurate results, the time must be measured with great precision. In the above example an error of one whole second in the time causes an error of nearly three-quarters of a degree in the latitude. With a stop-watch the time may be estimated to the tenth of a second, but it is evident that only approximate determinations of latitude are possible by this method with the instruments at the disposal of the surveyor.

The method is of interest, because it may be practised upon a star without the use of any Nautical Almanac Tables. It will give best results in high latitudes with observations made as near to the prime vertical as possible.

There are many other methods by which latitude may be determined, but for the most part they are not so convenient nor do they allow of the same elimination of instrumental errors as the four standard methods described. The following is an illustration of a method in which horizontal angles only have to be measured :—

Determination of latitude by the Measurement of the Horizontal Angle between Two Circumpolar Stars at their Greatest Elongations one on each Side of the Meridian.

Let c be the co-latitude, p_1 and p_2 the respective polar distances of the two stars, A_1 and A_2 the azimuths at elongation, one being measured to the East and the other to the West.

The measured angle $= A_1 + A_2$.

Then $$\sin p_1 = \sin c \sin A_1, \qquad (1)$$

and $$\sin p_2 = \sin c \sin A_2. \qquad (2)$$

(1) + (2) gives

$$2 \sin \frac{p_1 + p_2}{2} \cos \frac{p_1 - p_2}{2} = 2 \sin c \sin \frac{A_1 + A_2}{2} \cos \frac{A_1 - A_2}{2}.$$

(1) − (2) gives

$$2 \cos \frac{p_1 + p_2}{2} \sin \frac{p_1 - p_2}{2} = 2 \sin c \cos \frac{A_1 + A_2}{2} \sin \frac{A_1 - A_2}{2}.$$

Dividing one equation by the other gives

$$\tan \frac{p_1 + p_2}{2} \cot \frac{p_1 - p_2}{2} = \tan \frac{A_1 + A_2}{2} \cot \frac{A_1 - A_2}{2}.$$

Since $A_1 + A_2$ is known, this enables $A_1 - A_2$ to be computed. Hence A_1 is found.

Then $$\sin c = \frac{\sin p_1}{\sin A_1}.$$

Example, taken from Handbook of Instructions to South Australian Surveyors.

Observed horizontal angle 77° 45′ between Canopus and β Tri. Aus. at opposite elongations, polar distances 37° 22′ and 26° 57′.

$$\tan\frac{p_1 - p_2}{2} = \tan 5° 12' 30'', \quad . \quad . \quad 8{\cdot}9597747$$

$$\tan\frac{A_1 + A_2}{2} = \tan 38° 52' 30'', \quad . \quad . \quad 9{\cdot}9064310$$

$$18{\cdot}8662057$$

$$\tan\frac{p_1 + p_2}{2} = \tan 32° 09' 30'', \quad . \quad . \quad 9{\cdot}7984562$$

$$\tan\frac{A_1 - A_2}{2}, \quad . \quad . \quad . \quad . \quad . \quad 9{\cdot}0677495$$

$$\therefore \quad \frac{A_1 - A_2}{2} = 6° 40'$$

and $$\frac{A_1 + A_2}{2} = 38° 52' 30''$$

$$\therefore \quad A_1 = 45° 32' 30''$$

$$\sin p_1 = \sin 37° 22', \quad . \quad . \quad . \quad . \quad . \quad 9{\cdot}7831268$$

$$\sin A_1 = \sin 45° 32' 30'', \quad . \quad . \quad . \quad 9{\cdot}8535522$$

$$\cos \text{lat.}, \quad . \quad . \quad . \quad . \quad . \quad . \quad 9{\cdot}9295746$$

$\therefore$ latitude $= 31° 45' 20''$.

Table giving Values of *m* for Reduction of Circum-Meridian Observations.

$$m = \frac{2 \sin^2 \frac{t}{2}}{\sin 1''}.$$

The values of m are given in seconds of arc.

Value of t in Minutes of Time.	Additional Seconds of Time.					
	0	10	20	30	40	50
0	0·0	0·1	0·2	0·5	0·9	1·4
1	2·0	2·7	3·5	4·4	5·4	6·6
2	7·8	9·2	10·7	12·3	14·0	15·8
3	17·7	19·7	21·8	24·0	26·4	28·8
4	31·4	34·1	36·9	39·8	42·8	45·9
5	49·1	52·4	55·8	59·4	63·0	66·8
6	70·7	74·7	78·8	83·0	87·3	91·7
7	96·2	100·8	105·6	110·4	115·4	120·5
8	125·7	130·9	136·3	141·8	147·5	153·2
9	159·0	165·0	171·0	177·2	183·5	189·8
10	196·3	202·9	209·6	216·4	223·4	230·4
11	237·5	244·8	252·2	259·6	267·2	274·9
12	282·7	290·6	298·6	306·7	315·0	323·3
13	331·7	340·3	349·0	357·7	366·6	375·6
14	384·7	393·9	403·3	412·7	422·2	431·9
15	441·6	451·5	461·5	471·5	481·7	492·0
16	502·5	513·0	523·6	534·3	545·2	556·1
17	567·2	578·4	589·6	601·0	612·5	624·1
18	635·9	647·7	659·6	671·6	683·8	696·0

For intermediate values of t the corresponding values of m may be found by simple interpolation.

Examples.

1. At a place in latitude North, the true zenith distances of *a* Cephei (declination 61° 58′ 21·1″) is determined as 26° 54′ 28·3″ N. The zenith distance of *a* Aquilæ (declination 8° 29′ 22·7″) is found as 26° 34′ 27·5″ S. Find the latitude of the place.

Ans. 35° 03′ 51·5″.

2. In latitude 30° S. the times of transit of a star whose declination is 20° S. are observed across the prime vertical. If the direction of the prime vertical is in error by 1°, show that the measured interval of time will be too great by about 14 seconds.

3. An observation made in Antarctica on November 19th, 1912, gave the altitude of the sun's centre as 42° 07·8′, the temperature being 17° F. and the barometer reading 27·2 inches. Correct for refraction and parallax, and compute the latitude of the place, given that the sun's declination is 19° 21·6′ S.

Ans. 67° 14·7′ S.

4. The declination of the sun being 20° 39·9′ S., its meridian altitude is observed as 43° 17′. The correction for refraction and parallax being — 00·9′, determine the latitude of the place.

Ans. 67° 23·8′ S.

5. The sun is observed on the prime vertical, morning and afternoon, the times by watch being 7 hrs. 30 min. and 4 hrs. 14 min. The sun's declination is 17° 31′ 30″. Compute the latitude.

Ans. 37° 17′ 30″.

6. At a place in S. latitude the interval between the passages of Sirius across the prime vertical is 6 hrs. 9 min. 19½ sec. mean time. The mean readings of the bubble on striding level were 10 N. and 14 S., each division being = 20″. The declination of the star is 16° 35′ 33″ S. What was the latitude of the place of observation ?

Ans. 23° 20′ 17″ S.

7. The hour angle of Aldebaran (dec. 16° 20′ 15″ S.) when on the prime vertical was found to be 4 hrs. 35 min. 19·5 sec. What was the latitude of the place of observation ?

Ans. 39° 04′ 3″ S.

8. At a place in the Southern Hemisphere γ^2 Ceti (dec. 2° 51′ 22″ N.) was observed at equal altitudes of 48° 02′ 20″, and the interval in mean solar time between the two occurrences was 16 min. 12 sec. Required the latitude of the place.

Ans. 43° 50′.

9. Antares crossed the prime vertical at 13 hrs. 52 min. sidereal time. Find the latitude of the place of observation.

R.A. of Antares, ...	16 hrs. 23 min.
Dec. ,, ...	26° 13′ S.

Ans. 31° 54′ 49″ S.

10. The altitudes of a star when it crosses the meridian and prime vertical are respectively 65° and 10° (corrected). Find the star's declination and latitude of place.

Ans. Lat., 29° 58′ 39″.
Dec., 4° 58′ 39″ S. in S. lat.
or N. in N. lat.

11. The altitude of Sirius on the prime vertical is found to read 39° 48′. The declination of Sirius is 16° 35′ 20″ S. Find the latitude of the observing station. Allow for refraction.

Ans. Lat., 26° 30′ 1″ S.

12. At a place in South latitude the altitude of a star was observed at its upper and at its lower culminations, the altitude corrected for refraction at upper culmination being 60° 45′ 15″, and at lower culmination 10° 16′ 15″. Find the latitude of the place of observation and the declination of the star.

Ans. Lat., 35° 30′ 45″.
Dec. S., 64° 45′ 30″.

13. On the evening of 8th February, 1914, at a place in S. latitude, the magnetic bearing of β Hydri at its Western elongation was 185° 47′ 35″, and that of θ Argus at its Eastern elongation was 137° 24′ 42″.

Declination of β Hydri, ... 77° 44′ 29″ S.
,, θ Argus, ... 63° 56′ 36″ S.

Determine the latitude of the place and the magnetic variation.

Ans. Latitude, 36° 24′ 56″.
Variation, 9° 30′ 20″ E.

14. The altitude of Regulus at 10 hrs. 08 min. sidereal time was 46° 52′ 32″ (fully corrected). From the Nautical Almanac we find :—

R.A. of Regulus, ... 10 hrs. 03 min. 17 sec.
Declination of Regulus, ... 12° 26′ N.

What was the correct altitude when on the meridian ?

Ans. 46° 52′ 37·4″.

15. On 9th March, 1914, at a place South of Equator in 140° E. longitude the following altitudes of α Virginis (Spica) were observed near its meridian passage and their times taken with a chronometer keeping local mean time :—

Observed Altitudes.	Local Mean Times.
57° 40′ 36″,	2 hrs. 02 min. 18 sec. a.m.
44′ 34″,	05 min. 54 sec. ,,
48′ 40″,	10 min. 50 sec. ,,
50′ 10″,	15 min. 58 sec. ,,
49′ 30″,	22 min. 10 sec. ,,
46′ 40″,	27 min. 00 sec. ,,
42′ 35″,	31 min. 02 sec. ,,

The sidereal time at G.M.N., March 8th, is 23 hrs. 1 min. 22·91 sec.

R.A. of Spica = 13 hrs. 20 min. 41·4 sec.
Declination of Spica = 10° 43′ 00″ S.

Find the latitude of the place.

Ans. 42° 52′ 51″.

16. The declination of a star being 40° S., what are the latitudes of the places where its meridian altitude will be 80° ?

Ans. 50° or 30° S.

17. In south latitude two stars are observed on the meridian, one north and the other south of the zenith, the difference of zenith distances being found to be 13′ 03·45″ N., the declinations of the stars being 45° 38′ 37·48″ S. and 42° 44′ 04·63″ S. respectively.

Find the latitude.

Ans. 44° 17′ 52·8″.

18. A south circumpolar star was observed at equal intervals shortly before and after its elongation, when it was found to change its altitude from 44° 35′ to 47° 35′, during an interval of 19 min. 47 sec., by watch keeping correct mean time.

Find the polar distance of the star and the latitude of the place of observation.

Ans. 37° 20′ 30″.
Latitude = 33° 27′ 58.

19. At 6.10 p.m., local mean time, by watch on 15th September, 1907, in longitude 151° 06′ 30″ East, the magnetic bearing of σ Octantis was 170° 37′ 30″, the bearing of the referring mark being 72° 50′ 45″, and the observed altitude of the star was 34° 36′.

R.A. of Octantis, ...	19 hrs. 12 min. 48 sec.
Declination of Octantis, . .	89° 14′ 49″ S.
Sidereal time at G.M.N., Sept. 15th,	11 hrs. 33 min. 12 sec.
,, ,, Sept. 14th,	11 hrs. 29 min. 15 sec.

Find the latitude of the observer and the true bearing of the referring mark.

Ans. Latitude = 33° 54′ 19″.

20. On March 6th, 1914, the altitude of Polaris, when corrected for instrumental errors and refraction, is found to be 46° 17′ 28″, the mean time of observation being 7 hrs. 43 min. 35 sec. p.m. and the longitude of the place 37° W.

Sidereal time at G.M.N., March 6th,	22 hrs. 53 min. 29·8 sec.
R.A. of Polaris, March 6th, . .	1 hr. 27 min. 37·3 sec.
N. declination of Polaris, March 6th,	88° 51′ 8″

Find the latitude.

Ans. N. 46° 3′ 35″.

21. The observatory at Stockholm is in latitude 59° 20′ 33″ N., and that at the Cape of Good Hope in latitude 33° 56′ 3·5″ S. The declination of Sirius is 16° 35′ 22″ S. Find the altitudes of Sirius when on the meridian at Stockholm and at the Cape of Good Hope respectively.

Ans. 14° 04′ 05″ and
72° 39′ 18·5″.

22. The upper transit of a South circumpolar star was observed to occur at 7 hrs. 05 min. 28 sec. p.m. local mean time, and to reach its greatest

western elongation at 11 hrs. 44 min. 30 sec. p.m., when its observed azimuth was 33° 48′.

Find the latitude of the place of observation and the declination of the star. *Ans.* Latitude, 31° 02′ 52″ S.
Declination, 61° 32′ 11″ S.

23. On March 13th, 1911, at a place South of the Equator, in longitude 9¼ hours E., at 6 minutes before apparent noon, the altitude of the sun's lower limb was found to be 58° 04′ 20″, at which time clouds prevented further observation. The sun's declination at G.M.N., March 13th, is 3° 15′ 07·4″ S., and on March 12th 3° 38′ 41·8″ S.

Find the latitude of the place by reduction to the meridian, the sun's semi-diameter being 16′ 07″, its parallax 5″, and refraction 37″.

Ans. 35° 02′ 28″.

24. The altitudes of a star when it crosses the meridian and the prime vertical of a place are a and b. If l is the latitude of the place, show that

$$\cot l = \tan a - \sec a \sin b.$$

25. The meridian altitude of Altair is 51° 55′ 45″, its declination being 8° 34′ 34″ N. and the meridian altitude of β Pavonis is 52° 54′ 32″, its North polar distance being 156° 36′ 18″. Find the latitude of the place of observation.

Ans. 29° 30′ 15·5″ S.

26. At a place, south of the equator, the meridian zenith distances of the two stars γ^2 Norma and σ Scorpii were observed, the former to the south, the latter towards the north. The observed difference of the zenith distances was found to be 19′ 21″. Find the latitude of the place of observation.

Declination of γ^2 Norma, . . 49° 57′ 08·3″ South
,, σ Scorpii, . . 25° 23′ 31·2″ South

Another observer, stationed some distance to the north, found the difference of the zenith distances of these stars to be exactly the same. Determine his latitude also.

Ans. 37° 50′ 00·25″ and
37° 30′ 39·25″.

27. The mean altitude reading from four observations of Polaris was 51° 39′ 34·25″, the mean readings of the alidade level E., 5·5, O., 6·5, one division of level = 15″, mean chronometer time 7 hrs. 09 min. 54·8 sec., the chronometer keeping L.M.T. and being 3 min. 24 sec. fast. The longitude of station was 0 hr. 2 min. 9 sec. E. G.S.T. at G.M.N. on the day of observation was 13 hrs. 05 min. 34·1 sec. Declination of Polaris, 88° 45′ 50·8″; R.A. of Polaris, 1 hr. 22 min. 26 sec. Barometer, 30·27″. Thermometer, 42°. Compute latitude of place. (Example from "Topographical Surveying," by Major Close.)

Ans. 51° 23′ 34″.

CHAPTER X.

THE DETERMINATION OF TIME BY OBSERVATION.

In this chapter it is proposed to consider the principal methods available to the surveyor for the practical determination of the local mean or sidereal time by observation. Other methods have been devised, but the methods about to be described are those that have proved in practice to be the most convenient and satisfactory. Nearly all the ordinary time determinations of the surveyor are made by the second of the following methods, a convenient observation that may be carried out in the day light, and by which the time may be readily found with ordinary instruments with an error of not more than one or two seconds. One second of time will, of course, correspond to 15″ of hour angle.

First Method—By Meridian Transits.—We know that the local sidereal time at the instant that a star is on the meridian is measured by the R.A. of the star. Consequently, if we make the observation upon a star whose R.A. is known, by setting a theodolite up in the meridian and noting the time of transit of the star across the vertical wire, we have clearly a very simple way of finding the sidereal time at that instant and thus of determining the error of a watch or chronometer.

A similar observation may be made upon the sun, by noting the times of transit of the E. and W. limbs. The mean of these times will be the time of transit of the sun's centre, which takes place at *apparent* noon. From the Nautical Almanac we can find the equation of time for the given date, from which the mean time at the instant may be found. If only one limb be observed, then allowance must be made for the time occupied by the sun's semi-diameter in crossing the meridian, which is given in the Nautical Almanac on page 1 for each month.

EXAMPLE.—*On December* 1*st*, 1914, *at a place in longitude* 9 *hrs.* 45 *min. E.*, *the meridian times of transit of the E. and W. limbs of the sun across the vertical wire of a theodolite were taken with a watch supposed to keep the standard time of the meridian* 9 *hrs.* 30 *min. E. The observed times of transit being* 11 *hrs.* 32 *min.* 32·5 *sec. and* 11 *hrs.* 34 *min.* 52·5 *sec., determine the error of the watch.*

From the Nautical Almanac we find that at Greenwich apparent noon on December 1st, 1914, the equation of time, to be subtracted from apparent time, is 11 min. 6·47 sec., and that it is decreasing, the variation in 1 hour being 0·918 second.

Therefore, $9\frac{3}{4}$ hours before this—*i.e.*, at apparent noon in longitude 9 hrs. 45 min. E.—the equation of time will be 11 min. 6·47 sec. + $9\frac{3}{4}$ × 0·918 sec. = 11 min. 15·4 sec.

∴ L.M.T. at L.A.N. = 11 hrs. 48 min. 44·6 sec.

∴ Standard time at L.A.N. = 11 hrs. 33 min. 44·6 sec.

But the time of transit of the sun's centre—*i.e.*, the mean of the two observed times—was 11 hrs. 33 min. 42·5 sec.

Therefore, the watch was 2 seconds slow.

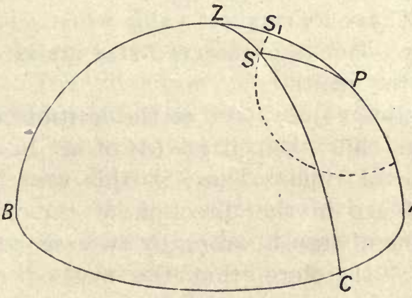


Fig. 42.

The Effect of an Error in the Direction of the Meridian.—If the instrument be in accurate adjustment, but the direction of the meridian be in error, then the meridian set out will pass through the zenith of the observer, but not through the celestial pole. In Fig. 42, let Z C denote the erroneous meridian, making an angle that we will call e with the true meridian Z P A. Then a star will intersect the apparent meridian at S, and the time noted will be either too soon or too late, according as the meridian is wrongly marked out to the East or West of the true direction, the error being measured by the hour angle S P Z, which we will call h.

$$P Z = c = \text{co-latitude}$$
$$P S = p = \text{polar distance of star.}$$

Then, in the triangle P Z S,

$$\cot p \sin c = \cot e \sin h + \cos c \cos h.$$

Since e and h are both small, we may write, without appreciable error, h and e instead of $\sin h$ and $\sin e$ respectively, and may put $\cos h$ and $\cos e$ each $= 1$.

$$\therefore \quad e (\cot p \sin c - \cos c) = h.$$

$$\therefore \quad h = e \frac{\sin (c - p)}{\sin p} . \quad . \quad . \quad . \quad (1)$$

Thus h will have its smallest value when p is nearly $= c$; that is to say, when the observed star makes its meridian transit near the zenith.

If in equation (1) $c = 60°$, or the latitude of the place is 30°, and $p = 40°$, then, if $e = 01'$ of arc, $h = 32''$ of arc or 2 seconds of time. Thus, in this case, an error of 1 minute of arc in the direction of the meridian will make the time of transit wrong by two seconds.

It is clear, therefore, that the method requires the meridian to be very accurately set out, and the instrument must be in perfect adjustment, if good results are to be obtained by this method.

In Fig. 42 we have illustrated the case where the star transits above the celestial pole. If the lower transit had been observed, then the angle h would be the supplement of the angle S P Z, and in this case the formula would become $$h = e \frac{\sin (c + p)}{\sin p}.$$

Both are included in the general formula,

$$h = e \frac{\sin \text{zenith distance}}{\sin p}, \text{ or } e \frac{\cos \text{alt.}}{\cos \text{dec.}},$$

which applies to all cases.

The error is thus very great if the polar distance of the star is small, and is least for those stars that transit near the zenith.

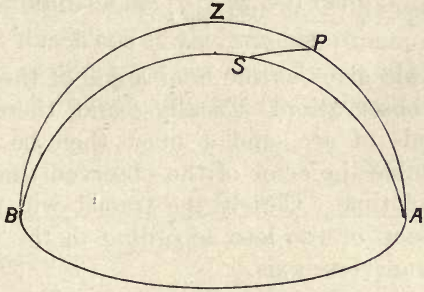


Fig. 42a.

The Effect of an Error in the Horizontality of the Transverse Axis. — The direction of the meridian may be accurately set out with the telescope horizontal or nearly so, and yet, if the transverse axis is not horizontal, the line of sight may depart considerably from the meridian at high altitudes. If the angle made by the transverse axis with the horizontal be determined by means of the striding level, the necessary correction to the time of transit may be made as follows :—

In Fig. 42a, the meridian actually swept out by a telescope with the transverse axis slightly tilted is represented by A S B, A and B being the North and South Points, and Z the zenith. The transit of the star is observed in consequence at a point S on this circle, and the error in time is measured by the angle S P Z.

In the triangle B P S

$$S P = p = \text{polar distance of star,}$$
$$B P = 180° - l = \text{supplement of latitude,}$$
$$\text{Angle } P B S = e = \text{error measured by striding level,}$$
$$\text{Angle } B P S = x = \text{required error in time of transit.}$$

$\therefore$ cot S P sin B P = cot e sin x + cos B P cos x.

$\therefore$ treating x and e as small quantities,

$$\cot p \sin l = \frac{x}{e} - \cos l.$$

$$\therefore \qquad x = e\,\frac{\sin (l + p)}{\sin p}, \text{ or } e\,\frac{\text{sin altitude}}{\text{cos dec.}}.$$

This formula gives us the hour angle of the star at the moment of observation. Usually e and, therefore, x will be in seconds of arc, and x must then be divided by 15 to determine the error of the observed time of transit in seconds of time. Clearly the transit will be observed either too soon or too late according to the direction of tilt of the transverse axis.

If the star transits below the pole, x will be the supplement of the angle B P S, and we get

$$x = e\,\frac{\sin (l - p)}{\sin p}, \text{ which again} = e\,\frac{\text{sin alt.}}{\text{cos dec.}}.$$

The error in time in this case increases with the altitude.

Example.—At a place in latitude 30° S. the sidereal time of transit of a star across the meridian is observed to be 12 hrs. 30 min. 17·5 sec., the declination of the star being 58° 30′ S. The readings of the striding level, one division of which = 13″, are :—

L.	R.
6·0	5·0
3·6	7·2
9·6	12·2
	9·6
	4) 2·6
	0·65

0·65 × 13 = 8·45″.

$$\therefore \text{ error in hour angle} = 8{\cdot}45 \times \frac{\sin 61^\circ\, 30'}{\sin 31^\circ\, 30'} = 14{\cdot}21''.$$

This is equivalent to 0·95 second of time.

As the right-hand side of the axis is the higher, and the telescope is directed towards the South, the transit is, therefore, observed too soon by this amount, and the corrected time of transit across the meridian is 12 hrs. 30 min. 18·45 sec.

Meridian Transits on Both Sides of the Zenith.—A considerable improvement may be made in the accuracy of the method by taking observations of the times of transit of two stars, one on each side of the observer's zenith.

In Fig. 43, let Z denote the zenith, P the celestial pole, A Z P B the direction of the true meridian, and C Z D the direction of the meridian actually set out, the figure being drawn as though the celestial sphere were viewed

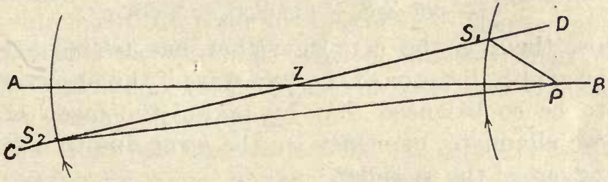


Fig. 43.

from above. Suppose that the times of transit of two stars are observed, one at S_1 and the other on the opposite side of the zenith as at S_2. Then, since both stars move in the same direction, as shown by the arrows, if the observed time of transit of S_1 is later than it should be, owing to the faulty determination of the meridian, the time of transit of S_2 will be correspondingly earlier. If the stars are well selected, it may be that the time errors of the two observations are equal and opposite, so that the mean of the two results will give a correct time determination in spite of the error in the setting out of the meridian. This will be the case if the hour angle $S_1 P Z$ is = the angle $S_2 P Z$, for then one observation will be just as much too soon as the other one is too late.

The conditions that this may be the case are readily obtained as follows :—

Let angle B Z D $= e =$ meridian error, and suppose that the hour angle $S_1 P Z = S_2 P Z = h$,

$c =$ co-latitude P Z.

Then, from the triangles $S_1 P Z$, $S_2 P Z$,

$$\frac{\sin h}{\sin e} = \frac{\sin Z S_1}{\sin P S_1} = \frac{\sin Z S_2}{\sin P S_2}.$$

But, since the error e is small, we may write very approximately $P S_1 = c - Z S_1$ and $P S_2 = c + Z S_2$.

$$\therefore \quad \frac{\sin (c - Z S_1)}{\sin Z S_1} = \frac{\sin (c + Z S_2)}{\sin Z S_2}.$$

$$\therefore \quad \sin c \,.\, \cot Z S_1 - \cos c = \sin c \cot Z S_2 + \cos c.$$

$$\therefore \quad \cot Z S_1 - \cot Z S_2 = 2 \cot c.$$

This, then, is the condition that has to be satisfied by the zenith distance of the two stars if the observations are to be so balanced that by taking the mean of the two we eliminate, or nearly so, the error due to a faulty setting out of the meridian.

The following table, based upon the above formula, gives the proper zenith distance of the star on the opposite side of the zenith to the pole, corresponding to different zenith distances of the other observed star, for different latitudes :—

Zenith Distance of Star on same Side as Pole.	Zenith Distance of Star on Opposite Side of Zenith to the Pole.							
	Lat. 10°.	Lat. 20°.	Lat. 30°.	Lat. 40°.	Lat. 50°.	Lat. 60°.	Lat. 70°.	Lat. 80°.
5°	5° 09′	5° 20′	5° 34′	5° 51′	6° 18′	7° 09′	9° 34′	85° 0′
10°	10° 39′	11° 26′	12° 29′	14° 30′	16° 55′	24° 22′	80° 0′	..
20°	22° 40′	26° 21′	32° 08′	43° 05′	70° 0′	..	..	..
30°	35° 56′	44° 53′	60° 0′	86° 55′	..	..	..	..
40°	50° 0′	65° 07′	87° 53′	..	..	..	..	..
50°	64° 04′	83° 39′	..	..	..	..	..	..
60°	77° 20′	..	..	..	..	..	..	..
70°	89° 21′	..	..	..	..	..	..	..

The advantage of selecting the two stars in this way may be illustrated by a computed example. Suppose that the place of observation is in latitude 30°, and that the polar distance of the star observed on the same side of the zenith as the pole is 40°, so that its zenith distance is about 20°. Suppose, further, that the marked meridian is as much as 1° in error.

Computing with these data the spherical triangle S P Z of Fig. 42, it may be shown that the hour angle S P Z is 2 min. 04·8 sec. In other words, the observed transit will take place too soon by this amount.

Now, according to the table, the star observed on the opposite side of the zenith should have a zenith distance of 32° 08′. Suppose it actually has a zenith distance of 32°, equivalent to a polar distance of 92°. Then, computing in the same way the hour angle of this star when on the faulty meridian, we find that its observed transit will be too late by 2 min. 04 sec.

Thus from one observation the chronometer would be set too fast by 2 min. 04 sec., and from the other it would be set too slow by about the same amount, and the mean of the two observations would give the time correct to the nearest second in spite of the fact that the direction of the meridian is 1° in error.

If, however, the zenith distances of the two stars are not balanced in the way indicated, the accuracy of the mean result is nothing like so great. If, for example, the two zenith distances were the same, the star observed on the opposite side of the zenith to the pole having a zenith distance of 20°, or a polar distance of 80°. Then, on computing the spherical triangle, it will be found that the observed transit of this star is too late by 1 min. 24 sec., so that the mean of the two observations is then in error to the extent of about 20 seconds.

Second Method—By Extra Meridian Observations of Sun or Star.—This is, as a rule, the most convenient and suitable method for the determination of time by the surveyor. It consists in the measurement of the altitude of sun or star when out of the meridian, at the same instant noting the chronometer time. Then, from a knowledge of the latitude of the place and the declination of the body observed we may compute the proper local time at the instant of observation, and so determine the error of the chronometer.

The most favourable time for making such an observation will be when the altitude of the celestial body is

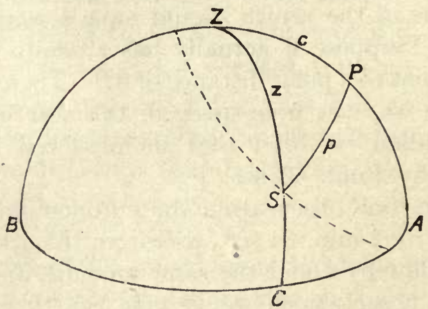


Fig. 44.

changing most rapidly, and this will be the case when it is near the prime vertical. This position has also other advantages, as we shall see in the course of the discussion.

As an altitude has to be measured, refraction must be allowed for, and as there is considerable uncertainty about this at low altitudes, the star observed should have an altitude of at least 15°.

The method involves the solution of the same spherical triangle that we have discussed in connection with extra-meridian observations for azimuth. Thus, in Fig. 44, if

S is the star observed, then in the spherical triangle Z P S we know the three sides :—

Z P = c = co-latitude,
S P = p = polar distance of star,
Z S = z = zenith distance, or the complement of the observed altitude.

Therefore, we can compute the hour angle S P Z, from which we can find the local sidereal time if we know the R.A. of the star, or this at once gives us the local apparent time in the case of the sun.

Let the angle S P Z $= h$.

Then, we have three available formulæ adapted to logarithmic computation, any one of which may be used for computing h. They are—

if $s = \frac{1}{2}(z + c + p)$

$$\sin \frac{h}{2} = \sqrt{\frac{\sin (s - c) \cdot \sin (s - p)}{\sin c \cdot \sin p}},$$

$$\cos \frac{h}{2} = \sqrt{\frac{\sin s \cdot \sin (s - z)}{\sin c \cdot \sin p}},$$

$$\tan \frac{h}{2} = \sqrt{\frac{\sin (s - c) \cdot \sin (s - p)}{\sin s \cdot \sin (s - z)}}.$$

The Choice of a Formula.—Of the three formulæ, that for $\cos \frac{h}{2}$ is somewhat the simplest, as we must find s in any case, and we have then only to find $s - z$ in addition. With the sine formula we have one more subtraction to make, but there is the advantage that only tables of log sines are used, and there is less risk of mistake in taking out the logarithms.

If, however, we are utilising the same observation, as may be done, for the determination of azimuth in addition, then we shall require to compute also the angle S Z P. In this case it is a decided advantage to select the tangent formula for the computation of both angles, for we shall then need only to look up four logarithms, as the same expressions sin s, sin $(s-c)$, sin $(s-p)$, and sin $(s-z)$ will occur in the tangent formulæ for both angles. If, on the other hand, we use the sine or cosine formula for the two angles, it will be necessary to look up six logarithms.

Another important point in the selection of a formula is this. The variation in value of the tangent of an angle, as the angle increases from 0° to 90°, is very much greater than in the case of a sine or cosine. Consequently a table of tangents will enable us to determine the value of an angle with greater precision than a table of sines or cosines. This is of practical importance when the angle under consideration is near to 0° or 90°. Thus there is very little variation in the value of the cosine of an angle up to 2° or 3°, and, if we wish to determine the values of such small angles to seconds, a table of cosines is not nearly so good as a table of tangents. Similarly, there is very little variation in the sine of an angle near to 90°, and it becomes difficult to compute such angles with precision from a sine table. It follows, therefore, that if h is near 0° or near to 90°, the tangent formula is the best one to adopt.

Data Necessary for Computation. — In addition to the measured altitude, we require a knowledge of the latitude of the place and the declination of the body observed. The declination for a star is taken straight from the Nautical Almanac, but the declination of the sun has to be found by using approximate values for the longitude and local time. If the result obtained shows that the assumed local time is very much out, the calculation should be repeated by using the corrected value of the local time found from the first computation.

Arrangement of the Computation.—It is worth some trouble to make a neat form for the computation. A good arrangement reduces the work, and is an aid to accuracy. The following, for instance, is the method adopted in the printed forms of the Queensland Survey Department :—

$p=$	$59° \ 34' \ 48''$	log sin	$=$	9·9356770
$c=$	$76° \ 05'$	log sin	$=$	9·9870611
$z=$	$66° \ 34' \ 19''$			19·9227381
2)	$202° \ 14' \ 07''$	subtract from		20·
$s=$	$101° \ 07' \ 03{\cdot}5''$	$\log \dfrac{1}{\sin p \sin c}$	$=$	0·0772619
$s-p=$	$41° \ 32' \ 15{\cdot}5''$	log sin	$=$	9·8215856
$s-c=$	$25° \ 02' \ 03{\cdot}5''$	log sin	$=$	9·6265032
			2)	19·5253507
$\therefore \ \frac{1}{2}h=$	$35° \ 22' \ 48''$	log sin	$=$	9·7626753

Where the same observation is to be utilised for both time and azimuth, a neat device is to proceed as follows :—

$\log \sin (s-c) =$	say	9·949960
$\log \sin (s-p) =$		9·046045
$\log \sin (s-z) =$		9·875721
		28·871726
subtract $\log \sin s$		9·945558
	2)	18·926168
		9·463084

From this we have simply to subtract $\log \sin (s-z)$ and $\log \sin (s-p)$ in order to get $\tan \frac{h}{2}$ and $\tan \frac{Z}{2}$ respectively.

	9·463084		9·463084
$\log \sin (s-z) =$	9·875721		9·046045
$\log \tan \frac{h}{2}$	9·587363	$\log \tan \frac{Z}{2}$	10·417039

Having Computed the Hour Angle to Find the Time of the Observation.—In the case of a star the angle S P Z, turned into time by dividing by 15, measures the interval of sidereal time after or before the time of culmination, according as the star is observed on the West or East of the meridian. But the R.A. of the star is equal to the sidereal time at the instant of culmination. Therefore, the sidereal time at the moment of observation is obtained by adding (or subtracting) the value of *h* to the R.A. of the star. This may be turned into mean time in the way already discussed.

Thus, if the R.A. of the star is 7 hrs. 30 min., and the angle *h* is 35°, the star being observed in the West, then the local sidereal time at the moment of observation is 7 hrs. 30 min. + 2 hrs. 20 min. = 9 hrs. 50 min.

If the sun has been observed, the value of the angle *h* at once gives us the interval of solar time before or after the meridian transit of the sun—that is to say, it gives us the local apparent time. To convert this into mean time the equation of time must be determined at that particular instant. To do this we first find the corresponding Greenwich apparent time, by allowing for the difference of longitude, and then take the equation of time from page 1 of the Nautical Almanac, allowing for the hourly variation.

Suppose, for example, that the angle *h*, for a sun observation, is 48° 20′, the observation being made at a place in longitude 60° W. on May 23rd in the afternoon. We have, therefore,

Local apparent time, . . .	3 hrs. 13 min. 20 sec.
Longitude, ...	4 hrs. 0 min. 0 sec.
Greenwich apparent time, May 23rd,	7 hrs. 13 min. 20 sec.

We have then to find the equation of time at this instant. The Nautical Almanac gives for this date, 1914, the equation of time at apparent noon, Greenwich, as 3 min. 30·40 sec. The variation in one hour is given as 0·191 second, the equation decreasing on successive days. The Almanac states that the equation of time is to be subtracted from apparent time.

Hence, at the given instant,

Equation of time = 3 min. 30·40 sec. − 7·222 × 0·191 sec. = 3 min. 29·02 sec.

Therefore, the required mean time is

3 hrs. 13 min. 20 sec. − 3 min. 29·02 sec. = 3 hrs. 09 min. 50·98 sec.

Averaging Several Observations of the Same Star. — In practice it is usual to take at least two, and commonly four, observations in as quick succession as possible, half being taken with F.L. and half with F.R. The computation is then made as though one observation only had been taken, the mean of the altitudes being assumed to be the true altitude at the mean of the noted chronometer times.

The object of this procedure is to eliminate instrumental errors, but this is done at the expense of introducing another error due to the fact that the assumption made is not mathematically exact. The investigation of the magnitude of the error thus introduced into the work is too complex for insertion here, but it may be stated that the surveyor is quite safe in thus averaging altitude observations extending over a range of 2° in altitude under ordinary conditions. The error thus made in an extra-meridian time determination is then generally only a small fraction of a second of time, its exact magnitude depending upon the latitude of the observer, the declination, and hour angle of the heavenly body. It is least when the hour angle is nearly 90°.

Observations on Both East and West Stars.—It is a great improvement in accuracy to take one set of observations upon a star in the east and another corresponding set, under as similar conditions as possible, upon a star in the West. The averaging of two such sets of observations tends to eliminate certain classes of errors, and this should always be done where the highest accuracy is sought. If, for example, the refraction assumed is too great, the corrected altitude will be too low, and the computed time will be too early for a star in the east, while it will be correspondingly too late for a star in the west. If the two errors are about equal, as will be the case if the E. and W. stars make about the same horizontal angle with the meridian, and are observed at about the same altitude, then the average of the two sets of results will be correct. Similarly, the effects of any systematic error in the measurement of altitude are eliminated by pairing sets of observations in this way. The same applies to extra meridian observations for azimuth.

Example of Extra Meridian Observation on Sun for Time.

Forenoon Observations.

Place—Survey Office, Adelaide.
Longitude—9 hrs. 14 min. 20 sec. E.
Latitude—34° 55′ 38″ S.
Value of 1 division of bubble—10″.

Thermometer—56°.
Barometer—30·49 inches.
Date—15th July, 1914.
Standard Meridian—9 hrs, 30 min. E.

Chronometer keeping approximately standard time.

Observations.

Observed Limb.	Face.	Vertical Angles. A.	Vertical Angles. B.	Vertical Angles. Mean.	Level. E.	Level. O.	Chronometer Time. H.	M.	S.
L	L	20° 08′ 50″	20° 09′ 10″	20° 09′ 00″	10	10	9	30	23
U	R	20° 49′ 10″	20° 49′ 00″	20° 49′ 05″	11	9	9	31	29
L	R	20° 27′ 20″	20° 27′ 00″	20° 27′ 10″	11·5	8·5	9	32	40
U	L	21° 07′ 20″	21° 07′ 50″	21° 07′ 35″	10	10	9	33	44
			Means, .	20° 38′ 12″	10·6	9·4	9	32	04

Computation for sun's declination at assumed approximate time of observation.

Approximate standard time of observation, 14/7/14, ...	21 hrs. 32 min. 04 sec.
Difference for standard meridian, . .	9 hrs. 30 min. 00 sec.
Corresponding G.M.T., . .	12 hrs. 02 min. 04 sec.
Declination: 14th July, 1914 (G.M.N.), .	21° 47′ 03·3″
Difference for 12 hrs. 02 min. 04 sec., .	04′ 28·6″
Declination at instant of observation (North),	21° 42′ 34·7″
Sun's South Polar Distance, . . .	111° 42′ 34·7″

Formula— $$\tan\frac{h}{2} = \sqrt{\frac{\sin(s-c)\sin(s-p)}{\sin s \sin(s-z)}}.$$

CALCULATION.

Mean of observed altitudes, . . .	20° 38′ 12″
Level correction, ...	6″
	20° 38′ 06″
Refraction and parallax, ...	2′ 21″
Corrected altitude, ...	20° 35′ 45″
Zenith distance $= z$, ...	69° 24′ 15″
Co-latitude $= c$, ...	55° 04′ 22″
Sun's polar distance $= p$, . . .	111° 42′ 35″
$2s$, ...	236° 11′ 12″
s, ...	118° 05′ 36″
$s - c$, ...	63° 01′ 14″
$s - p$, ...	6° 23′ 01″
$s - z$, ...	48° 41′ 21″
$\log\sin(s-c)$, ...	9·949960
$\log\sin(s-p)$, ...	9·046045
$\log\operatorname{cosec} s$, ...	10·054442
$\log\operatorname{cosec}(s-z)$, ...	10·124279
$\log\tan^2\frac{h}{2}$, ...	19·174726
$\log\tan\frac{h}{2} = \tan 21° 08′ 28″$, . . .	9·587363
h, ...	42° 16′ 56″
h (in time), ...	2 hrs. 49 min. 08 sec.
Local apparent time $= 24$ hrs. $- h$, . .	21 hrs. 10 min. 52 sec.
Longitude, ...	9 hrs. 14 min. 20 sec.
Greenwich apparent time, . . .	11 hrs. 56 min. 32 sec.

Equation time at G.A.N., . . .	5 min. 33 sec.
Correction for 11 hrs. 56 min. 32 sec., .	3 sec.
Equation time instant observation, . .	5 min. 36 sec.
L.A.T., ...	21 hrs. 10 min. 52 sec.
L.M.T., ...	21 hrs. 16 min. 28 sec.
Diff. Standard Merid., ...	15 min. 40 sec.
Local Standard time, ...	21 hrs. 32 min. 08 sec.
Chronometer time, ...	21 hrs. 32 min. 04 sec.
Error of Chronometer, ...	04 sec. slow

EXAMPLE FOR REDUCTION.

With the same instrument as that used in the preceding observation a similar set of four sun observations was taken on the afternoon of July 21st, 1914, at the same place. The mean altitude obtained was 23° 53′ 36″, the average alidade level readings were E. 10·5, 0, 9·5. The mean of the chronometer times was 2 hrs. 52 min. 52·5 sec.

From the Nautical Almanac—

Declination of sun, at G.M.N., July 20th, 1914,	20° 47′ 18·2″ N.
Variation in one hour at noon on the 20th, .	27·60″
,, ,, ,, 21st, .	28·47″
Equation of Time, G.A.N., July 20th (to be added to apparent time), ...	6 min. 05·99 sec.
Variation in one hour, ...	0·165 sec.

Longitude, standard time, and latitude are given in the preceding case. The chronometer being supposed to keep standard time, determine its error. *Ans.* 02·1 sec. slow.

The Effect of an Error in Latitude.—It is important that we should know to what degree of precision the latitude must be known in order that the time may be determined. This may be readily investigated in a manner similar to that adopted with corresponding problems previously.

From the spherical triangle S Z P of Fig. 44,

$$\cos z = \cos c \cos p + \sin c \sin p \cos h.$$

If c is too *large* by a small amount y, then, for the same measured zenith distance z, h will be too *small* by an amount x, and we shall have

$$\cos z = \cos (c + y) \cos p + \sin (c + y) \sin p \cos (h - x).$$

Subtracting these two equations, and treating x and h as small quantities, we readily get

$$x = y \frac{-\cos c \cos h \sin p + \sin c \cos p}{\sin c \sin p \sin h}$$

$$= y \frac{\cot Z}{\sin c},$$

where Z denotes the azimuth angle S Z P.

This shows that x will be very large compared with y, if Z is nearly equal to 0, or if c is nearly 0. That is to say, a small error in the latitude will produce a very large error in the time if the body is observed near to the meridian, or if the observation is made in high latitudes near to either terrestrial pole.

On the other hand, if Z is 90°—*i.e.*, if the observation is made on the prime vertical—x is 0, and an error in latitude makes no difference. In this case the angle S Z P is a right-angled triangle, and we can get a relation between p, z, and h that does not involve c at all, so that a knowledge of the latitude is unnecessary. If the observation is made near to the prime vertical, therefore, an error in latitude will produce very little effect on the time determination.

The following table, based upon the above formula, gives the error in time corresponding to an error of 1′ in the latitude for different azimuth angles :—

Error in Time corresponding to 1′ Error in Latitude.*

Azimuth of Observed Body.	Latitude of Place.				
	0°.	30°.	40°.	50°.	60°.
	Seconds.	Seconds.	Seconds.	Seconds.	Seconds.
45°	4·0	4·6	5·2	6·2	8·0
60°	2·3	2·6	3·0	3·5	4·5
80°	0·7	0·8	0·9	1·1	1·4
90°	0·0	0·0	0·0	0·0	0·0

* If the word *Declination* be substituted for *latitude*, the same table will give the error in time due to an error of 1′ in the Declination, the first column representing, not the azimuth, but the angle Z S P.

This all points to the desirableness of making the observation as near to the prime vertical as possible.

The Effect of an Error in the Measured Altitude.—By a method similar to that adopted in the last paragraph it may be readily shown, if x is the error in the hour angle corresponding to an error y in the observed altitude, that

$$x = y \operatorname{cosec} Z \operatorname{cosec} c$$

x clearly becomes very great if either Z or c are small, and it has its least value when Z and c are each 90°. Thus, again, an error of observation has the least effect when the observation is made on a celestial body near the prime vertical, and the most favourable place for making the observation is at the equator.

TABLE SHOWING ERROR IN TIME DETERMINATION OWING TO AN ERROR OF 1′ IN THE MEASURED ALTITUDE, WITH DIFFERENT AZIMUTHS OF THE OBSERVED BODY.

Azimuth of Observed Body.	Latitude of Place.				
	0°.	30°.	40°.	50°.	60°.
	Seconds.	Seconds.	Seconds	Seconds.	Seconds.
45°	5·6	6·4	7·3	8·7	11·3
60°	4·6	5·3	6·0	7·1	9·2
80°	4·1	4·7	5·3	6·3	8·1
90°	4·0	4·6	5·2	6·2	8·0

This table deserves a little careful consideration, as it shows the degree of precision with which altitudes must be measured if the time is to be determined within one second. Under the most favourable possible conditions an error of $\frac{1}{4}$ minute of arc will cause an error of one second in the time, and it may produce an error of two seconds or even more.

EXAMPLE.—In the extra-meridian observation for time set out at length in paragraph just preceding show that an error of 1′ in the measured altitude will produce an error of 7 seconds in the time.

The Effect of an Error in the Declination of the Sun caused by a Defective Knowledge of Longitude or Local Time.—With star observations the Nautical Almanac gives us the declination of the star with all the precision that is required, but with sun observations the surveyor has first of all to compute the declination. To do this he requires to know both his longitude and the approximate local mean time.

From the formula

$$\cos z = \cos c \cos p + \sin c \sin p \cos h$$

it appears that the relation between an error in p and an error in h will be of precisely the same nature as the relation between an error in c and an error in h. So that if x denotes the error in the hour angle corresponding to an error y in the declination

$$x = \frac{\cot \mathrm{Z\,S\,P}}{\sin p} \cdot y.$$

Thus the table already given, showing the error in time caused by 1′ error in latitude, also gives the error in time caused by 1′ error in declination, provided that the first column is taken as representing the angle Z S P instead of the azimuth.

We have already seen that the maximum rate of variation of the declination of the sun is a little less than 1′ per hour. So that to get the declination of the sun to the nearest minute it is sufficient to know the time to the nearest hour. But one hour of time corresponds to 15° of longitude, so that it is seldom that the surveyor will not know his longitude sufficiently well for this purpose.

It will be seen from the table that, in order to determine the time to the nearest second, it will be necessary to know the declination within only about one-fifth of a minute of arc under almost the worst conditions of observation considered in the table. For this it will be usually sufficient to know the local time within a quarter of an hour.

If the local time is not known with sufficient accuracy, its value must be assumed for the purpose of finding the approximate declination. This is then used in a preliminary calculation made to determine the time. The calculation is then made over again, using the approximate local time so found in order to get a more accurate value of the sun's declination, which in turn is used in the computation to obtain a more accurate determination of the local mean time.

Third Method—By Equal Altitudes.—If a star be observed at the same altitude on opposite sides of the meridian, the two observations must clearly be made at equal intervals of time before and after the star's meridian

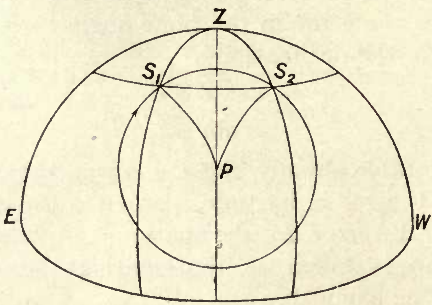


Fig. 45.

transit. Thus, in Fig. 45, if the star be observed in the two positions, S_1 and S_2, so that the zenith distances $Z\,S_1$ and $Z\,S_2$ are equal, then, if P is the celestial pole, the two hour angles $Z\,P\,S_1$ and $Z\,P\,S_2$ must be equal.

It follows that the mean of these two observed times is the time of the star's meridian transit. But the local sidereal time at the instant of the star's meridian transit is determined by the star's R.A., which is given by the Nautical Almanac. This local sidereal time may be reduced to mean time, and a comparison of this with the average of the two observed chronometer times determines the error of the chronometer.

With stars the method is capable of giving very accurate results, and it has the great advantage that no knowledge is required of latitude, declination, or even azimuth, and errors of graduation of the instrument have no effect upon the result. But to the surveyor it has the obvious drawback that a considerable interval of time must elapse between the observations.

As the accuracy of the determination depends upon the altitude being the same at the two observations, the star should have an altitude of something more than 45°, in order to get rid of the uncertainties of refraction near the horizon.

Example.—On September 1st, 1914, β Crucis was observed East of the meridian at 10 hrs. 42 min. 30·5 sec. by a chronometer keeping sidereal time. It was again at the same altitude West of the meridian at 14 hrs. 51 min. 20·7 sec. Find the error of the clock.

East	10 hrs. 42 min. 30·5 sec.
West	14 hrs. 51 min. 20·7 sec.
	2) 25 hrs. 33 min. 51·2 sec.
Meridian transit by chronometer, .	12 hrs. 46 min. 55·6 sec.
R.A. of star, ...	12 hrs. 42 min. 41 sec.
Chronometer correction, . . .	4 min. 14·6 sec.

As the chronometer is too fast, the correction is to be subtracted from the chronometer reading.

If, as is more usual, the chronometer keeps local mean time, the sidereal time at the meridian transit of the star must be reduced to local mean time in order to compare with the chronometer time. This cannot be done without a knowledge of the longitude.

Example.—At a place in longitude 8 hrs. 35 min. 27 sec. East, on the evening of September 1st, 1914, the star α Pavonis is observed East of the meridian at 7 hrs. 9 min. 20·5 sec., with a watch keeping local mean time. It is again observed at the same altitude to the West of the meridian at 9 min. 30·2 sec. after midnight. Find the error of the watch, having given

G.S.T. at G.M.N., September 1st, 1914,	10 hrs. 39 min. 13·38 sec.
R.A. of α Pavonis, ...	20 hrs. 18 min. 57·4 sec.

Ans. 8·1 seconds slow.

It is desirable, in order to make the determination as precise as possible, that a series of observations should be made upon the star on each side of the meridian, instead of one observation only. A few times should be taken when the star is on the East of the meridian at altitudes differing by 20 or 30 minutes of arc. A corresponding series of times should then be taken when the star is on the West of the meridian at the same altitudes. Since all that we want to ensure is that the altitude is the same at corresponding observations East and West of the meridian, there is no particular object in reversing the face of the instrument. The whole set of observations may be taken with the one face.

The Error due to a Slight Inequality in the Altitudes of two Corresponding Observations.—If in Fig. 45 Z S_1 = zenith distance of the first observation $= z$,

$$\begin{aligned} \mathrm{Z\,P} &= \text{co-latitude} = c \\ \mathrm{P\,S_1} &= \text{polar distance} = p \\ h &= \text{hour angle } \mathrm{Z\,P\,S_1} \\ \mathrm{Z} &= \text{angle } \mathrm{S_1\,Z\,P} = \text{azimuth of star} \end{aligned}$$

$$\cos z = \cos c \cos p + \sin c \sin p \cos h, \qquad . \qquad . \qquad (1)$$

Suppose now that at the second observation the zenith distance, instead of being z, is $z + y$, being in error by a small amount y. Then the hour angle Z P S_2 will be in error by a corresponding amount x, so that instead of being h, it will be $h + x$. Then, from the spherical triangle Z P S_2,

$$\cos (z + y) = \cos c \cos p + \sin c \sin p \,.\, \cos (h + x). \qquad (2)$$

Subtracting (2) from (1), treating x and y as small quantities, we get

$$y \,.\, \sin z = x \sin c \sin p \sin h.$$

But
$$\frac{\sin z}{\sin h} = \frac{\sin p}{\sin \mathrm{Z}},$$

$$\therefore \qquad x = \frac{y}{\sin c \sin \mathrm{Z}}.$$

We see thus that the error x in the hour angle, corresponding to an error y in the second altitude, will be least when $Z = 90°$, and will be greater the smaller the value of Z. We draw, therefore, the practical conclusion that the observations are best made on stars near the prime vertical.

If the declination of a star is slightly less than the latitude, it will cross the prime vertical near the zenith and the interval between the times of transit will be small. This, therefore, is a convenient observation to make, and the conditions are favourable to accuracy.

The Determination of Time by Equal Altitudes of the Sun.—The above method is an extremely simple one as applied to the stars, because the declination of a star remains constant during the period over which the observations extend. But in the case of the sun the declination changes so rapidly that it cannot be considered as constant, and the theory becomes complicated by the fact that allowance must be made for the alteration of declination in the interval between the observations. Referring again to Fig. 45, if p denotes the polar distance of the sun when it is on the meridian, then at the first sight, when the sun is at S_1, the polar distance will be $p \pm y$, and at the second sight, when the sun is at S_2, the polar distance will be $p \mp y$. The $+$ or $-$ sign is to be taken in the first of these expressions according as the sun is approaching or leaving the elevated pole.

If p were constant, we should have

$$\cos z = \cos p \cos c + \sin p \sin c \cos h.$$

But if at the first observation, S_1, the polar distance is $p + y$, the hour angle will be $h + x$, and we have

$$\cos z = \cos (p + y) \cos c + \sin (p + y) \sin c \cos (h + x).$$

Subtracting these two equations, and treating x and y as small quantities, we get

$$O = y \sin p \cos c - y \cos p \sin c \cos h + x \sin h \sin c \sin p.$$

$$\therefore \qquad -x = y\,(\cot c \operatorname{cosec} h - \cot p \cot h).$$

Under these conditions the first observation will be made when the sun is at an hour angle $h + a$ *before* apparent noon, where x is given by the preceding expression, and it may be positive or negative according as cot c cosec h is $<$ or $>$ cot p cot h.

Similarly the second observation will be made with the sun at an hour angle $h - x$ *after* apparent noon, and it may be shown in the same way as before that the value of x is given in this case also by the same mathematical expression.

The mean of these two observed times will therefore be when the sun is at an hour angle x before apparent noon.

When the sun is leaving the elevated pole, instead of approaching it, the mean of the two observed times will be when the sun is at an hour angle x after apparent noon.

Thus, the true time of transit—*i.e.*, the time of apparent noon—is given by

$$\text{Mean of observed times} \pm \tfrac{1}{15}\, y\, (\cot c \operatorname{cosec} h - \cot p \cot h).$$

y is the alteration in the sun's declination in half the time interval between the two observations.

h is half the time interval between the two observations reduced to angular measure.

The $+$ sign is to be taken if the sun is *leaving* the elevated pole, and the $-$ sign when it is *approaching* the elevated pole.

Just as with star observations, it is necessary, in order to obtain the best results, that a series, say four or six, of observations should be taken to the sun in the forenoon and a corresponding set in the afternoon, the sights in each case being taken alternately to the upper and lower limbs.

EXAMPLE.—At Adelaide, longitude 9 hrs. 14 min. 20 sec. E., latitude 34° 55′ 38″ S., on July 21st, 1914, equal altitude observations of the sun were taken in the forenoon and afternoon. The means of the noted times were 9 hrs. 35 min. 03 sec. a.m. and 2 hrs. 37 min. 15 sec. p.m. by a watch keeping mean time.

	12 hrs. 00 min. 00 sec.	
subtract	9 hrs. 35 min. 03 sec.	
	2 hrs. 24 min. 57 sec.	
add	2 hrs. 37 min. 15 sec.	
2)	5 hrs. 02 min. 12 sec.	= time between observations.
	2 hrs. 31 min. 06 sec.	∴ $h = 37° 46' 30''$.
subtract from	2 hrs. 37 min. 15 sec.	
	0 hr. 6 min. 09 sec.	= time by watch at apparent noon.

c	$= 55° 04' 22''$
Declination at G.A.N., July 21st, . .	$20° 36' 02{\cdot}5''$
Correction for longitude, ...	$2' 41{\cdot}5''$
Declination at L.A.N., ...	$20° 38' 44''$
∴ p, ...	$110° 38' 44''$
cot c . cosec h	$= 1{\cdot}140$
cot p cot h	$= -\ {\cdot}486$
cot c cosec h — cot p cot h . .	$= 1{\cdot}626$

Change in declination in 2 hrs. 31 min. 06 sec. = $71{\cdot}69''$, and sun is *approaching* elevated pole,

$$\therefore \text{ time of apparent noon } = 6' \, 09'' - \frac{1{\cdot}626 \times 71{\cdot}69}{15} \text{ seconds}$$

$$= 6' \, 09'' - 7{\cdot}6'' = 6' \, 01{\cdot}4''.$$

But, from the Nautical Almanac, the equation of time to be added to apparent time at L.A.N. is $6' \, 08{\cdot}3''$, which is, therefore, the true time of apparent noon.

Thus the watch is 7 seconds slow.

Fourth Method—Almucantar Method for Time Observations.—In 1884 Mr. S. C. Chandler, at the Harvard College Observatory, U.S.A., devised a form of instrument in which the telescope was fixed at a constant angle with the vertical, so that the line of sight traced out a horizontal circle on the celestial sphere, and observations for the determination of latitude and other purposes were made by noting the times of transit of stars across the fixed horizontal circle. The instrument was named an "almucantar," and it proved to be capable of very remarkable work. The same principle may be readily applied with an ordinary theodolite, and experience has shown that extremely accurate determinations of time are possible in this way.*

Any horizontal circle may be used for the observations, but the most convenient is the one that passes through the pole of the observer. This has been named the "co-latitude circle," its zenith distance being everywhere equal to the co-latitude. The formulæ for reduction then become very simple. The method consists in observing the times of transit of a series of East and West stars, somewhere near the prime vertical, across the horizontal

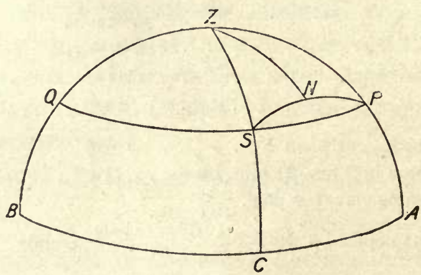


Fig. 45*a*.

wire of a telescope that is set to an altitude equal to that of the pole. Allowance must be made for refraction, and, therefore, the telescope is actually set so that its altitude as read off on the vertical circle is equal to the latitude of the place plus refraction.

In Fig. 45*a*, Z denotes the zenith, P the celestial pole, A and B the North and South points, P S Q the co-latitude circle. Let S denote the position of a star, somewhere near the prime vertical, as it crosses the co-latitude circle.

* See paper by W. E. Cooke, "On a New and Accurate Method of determining Time, Latitude, and Azimuth with a Theodolite"—Monthly Notices, Royal Astronomical Society, January, 1903.

Let $ZP = c =$ co-latitude.

$PS = p =$ star's polar distance, measured, of course, along the great circle arc PNS and not along the small circle PSQ.

Angle $SPZ = h =$ hour angle of star.

Angle $SZP = Z =$ azimuth of star measured from elevated pole.

Then, since $ZS = c$, ZSP is an isosceles triangle, and, if ZN be drawn perpendicular to the great circle arc joining S and P, it will divide SZP into two equal right-angled triangles.

From the triangle ZNP

$$\cos NPZ = \tan PN \cot ZP$$

$$\therefore \qquad \cos h = \tan \frac{p}{2} . \cot c = \tan \frac{p}{2} . \tan l \quad . \quad . \quad (1)$$

if l is the latitude of the place.

To determine the azimuth at which a star will cross the co-latitude circle, from the same triangle

$$\cos ZP = \cot NZP \cot NPZ.$$

$$\therefore \qquad \cos c = \cot h . \cot \frac{Z}{2},$$

$$\text{or } \cot \frac{Z}{2} = \sin l . \tan h. \quad . \quad . \quad (2)$$

Formula (1) enables the time of transit to be computed, and formula (2) gives the azimuth if required.

If an observation on one star in the East is balanced by a corresponding observation on a star in the West of somewhere about the same declination, then the mean of the two time observations will give a correct result even if the co-latitude circle is considerably out. If, for instance, the co-latitude circle is set out too low, the observed time of transit in the East will be too soon, but that in the West will be too late, and if there is not much difference in the declinations of the stars the time of transit will be just as much too soon in the one case as it is too late in the other. Thus by averaging the two results any small error in the setting out of the co-latitude circle is practically eliminated, and it is not necessary, therefore, in order to apply the method that the latitude of the place should be known with precision. An approximate latitude will suffice.

For precisely the same reasons as have been investigated when dealing with extra-meridian observations for time, slight errors in latitude, declination, and altitude will have least effect upon the result when the stars observed are near the prime vertical. The stars should be selected from a zone of about 20° on each side of the prime vertical.

EXAMPLE.—*On May 3rd,* 1903, *in Lat.* 31° 56′ 45″ *S., the transit of β Orionis was observed in the West across the co-latitude circle at* 8 *hrs.* 55 *min.* 1·5 *sec. by a watch keeping sidereal time. The transit of α Virginis was similarly observed in the East at* 9 *hrs.* 20 *min.* 23·4 *sec. Determine the error of the watch.*

	β Orionis.	α Virginis.
Declination,	8° 19′ 2·7″ S.	10° 39′ 30·1″ S.
p,	81° 40′ 57·3″	79° 20′ 29·9″
$\frac{1}{2} p$,	40° 50′ 28·6″	39° 40′ 15″
log tan $\frac{p}{2}$,	9·9367323	9·9187412
log tan l,	9·7948752	9·7948752
log cos h,	9·7316075	9·7136164
h,	57° 22′ 58″	58° 51′ 31″
h in time,	3 hrs. 49 min. 32 sec.	3 hrs. 55 min. 26 sec.
R.A. of star,	5 hrs. 09 min. 52·6 sec.	13 hrs. 20 min. 07·5 sec.
Computed time,	8 hrs. 59 min. 24·6 sec.	9 hrs. 24 min. 41·5 sec.
Observed time,	8 hrs. 55 min. 01·5 sec.	9 hrs. 20 min. 23·4 sec.
Error of watch (slow),	4 min. 23·1 sec.	4 min. 18·1 sec.

Mean determination of watch error. — 4 min. 20·6 sec. slow.

Adjustment of Telescope during Observation.—It is the most essential thing for accurate work, in observations of this kind, that the telescope should throughout make exactly the same angle with the horizontal. It is not of such importance that the altitude should be exactly equal to the latitude, as it is that the altitude should remain the same throughout the observations. Now, no matter how carefully a transit theodolite is adjusted, the bubble attached to the vertical circle will not remain precisely in the centre of its run as the telescope is turned from star to star. It is, therefore, essential to accurate work that this bubble should be adjusted to the centre of its run just before the star crosses the horizontal wire in each case. This must be done, of course, by the adjusting screw on every transit theodolite that moves both telescope and vertical circle together without affecting the altitude reading. After the reading on the vertical circle has been set for the first star so that the altitude is equal to the latitude plus refraction, the altitude screw which would alter this reading must on no account be touched. But at each observation the horizontal line of the vertical circle must be adjusted without altering the reading of the vernier.

To get the most accurate results observations must be made upon a number of stars, at least six in the East and six in the West, and the mean of all the determinations is taken. The East and West stars should be selected so that the angles in azimuth that one set make to the East are as nearly as possible equal to the angles that the other set make to the West.

Sun Dials.

Whilst the sun dial does not provide the surveyor with a means of determining local time with anything like the precision obtainable by the methods that have been described, it enables the time to be fixed quite sufficiently near for the regulation of watches and clocks for ordinary purposes, and the instrument may be read just as easily as a clock. It is especially useful in the remote parts of sparsely populated countries where no other means of checking the clock times are available.

When a sun dial is illuminated by the direct light of the sun the shadow of a straight line or sharp straight edge is thrown upon a plane containing a graduated circle so marked that the apparent solar time is indicated by the reading at the place where the shadow intersects the circle. The plane containing the graduated circle may be either horizontal, vertical, or inclined. The straight edge, the shadow of which is thrown upon the circle, is always set up so as to be parallel to the earth's axis. It is called the *stile*, or *gnomon* of the dial. When the graduated circle or "plane of the dial" is horizontal we have what is known as a *horizontal dial*, and as this is the most common form we will consider it first.

The Horizontal Dial.—In Fig. 46, let M B L A represent the plane of the dial, which we may suppose to be extended indefinitely so that M B L A is the circle in which it intersects the celestial sphere. C P is the direction of the gnomon, which again we may suppose to be produced to intersect the celestial sphere in the celestial pole P. B P A is the plane of the meridian.

If now S denotes the position of the sun, the line of intersection of the shadow of the gnomon C P with the plane of the dial will be the line of intersection of the plane containing C P and S with the plane M B L A. M P L represents in the figure the plane passing through S and C P, and M C L is the line of intersection of this plane with the plane of the dial, or C L is the direction of the shadow of the gnomon.

Neglecting the slight alteration in the declination of the sun during the hours of daylight, S will describe a circle uniformly on the celestial sphere about P as centre. The angle S P B is the hour angle of the sun, decreasing or increasing uniformly with the time according as the observation is made in the morning or in the afternoon.

Then in the right-angled triangle L P A

$$A\,P = l = \text{latitude of place.}$$

$$\text{Angle } A\,P\,L = h = \text{hour angle of sun.}$$

$$A\,L = x = \text{required division along the dial corresponding to hour angle } h.$$

$$\therefore \quad \sin l = \cot h \tan x, \text{ or } \tan x = \sin l \tan h.$$

Thus, to graduate the dial for the hourly intervals before and after noon, we must put $h = 15°$, $30°$, $45°$,

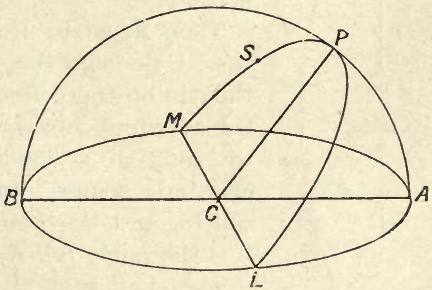


Fig. 46.

etc., in succession and compute the corresponding values of x, knowing, of course, the value of l.

Thus, if the latitude of the place is 30°, the first hourly division on each side of noon will be marked out at an angle with C A given by

$$\log \tan x = \log \sin 30° + \log \tan 15°,$$

from which $x = 7°\ 38'$.

The next hourly division, indicating either 10 a.m. or 2 p.m. will make an angle with C A given by

$$\log \tan x = \log \sin 30° + \log \tan 30°,$$

from which $x = 16°\ 6'$, and so on.

The reading of the shadow of the gnomon gives the local apparent time which must be corrected by the equation of time, as given by the Nautical Almanac, in order to obtain the mean time. A table of corrections may easily be drawn out for different times of the year.

The Prime Vertical Dial.—In this case the plane of the dial lies in the prime vertical. In Fig. 47 let A L B M be the plane of the dial, which we will again suppose is continued on indefinitely, so as to cut the celestial sphere. C P, the direction of the stile or gnomon, is again parallel to the earth's axis, but this time P will be the celestial pole below the visible horizon. A P B is the plane of the meridian.

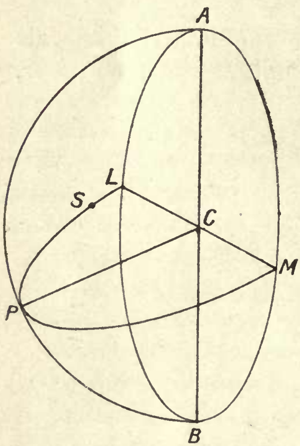


Fig. 47.

Then if, as in the previous case, S denotes the position of the sun on the celestial sphere, the apparent movement of S is to describe a circle on the celestial sphere with P as centre, and the hour angle of S is the angle S P A.

The shadow of P C thrown by S upon the plane of the dial will be C M, the line of intersection of the plane passing through S and P C with the plane of the dial.

In the right-angled spherical triangle P B M

$$P\,B = 90° - l = \text{co-latitude.}$$

$$\text{Angle } B\,P\,M = h = \text{hour angle of sun.}$$

$B\,M = x =$ required division along the dial corresponding to the hour angle h.

$\therefore \qquad \cos l = \cot h \tan x$

or $\qquad \tan x = \cos l \tan h,$

and by this formula the dial may be graduated in a similar manner to the horizontal dial.

Oblique Dials.—If the plane of the dial is inclined to the horizontal the dial is said to be "oblique." There is one case that is particularly simple, and has given rise to some of the simplest sun dial constructions. This is the case in which the plane of the dial is tilted so as to be perpendicular to the stile, so that it coincides with the plane of the celestial equator. With this arrangement the shadow of the stile on the dial moves round uniformly with the revolution of the sun and the hour divisions on the dial are consequently uniformly spaced.

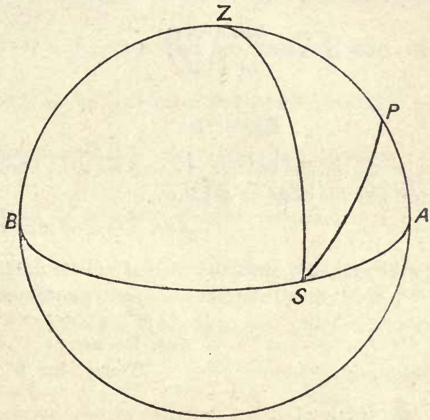


Fig. 48.

Time of Rising or Setting of a Celestial Body.

This is not of much value for the determination of time, because of the uncertainty of refraction on the horizon. In Fig. 48, if A S B be the plane of the horizon, Z the zenith, P the celestial pole, and S the body, which is exactly on the celestial horizon, then the spherical triangle P S A is right-angled at A, and

$$\cos S P A = \cot S P \tan P A.$$

$$\therefore \qquad \cos (\text{hour angle } S P Z) = -\tan \text{dec.} \tan \text{lat.}$$

From this the hour angle of the body at rising or setting may be computed, and this will determine the apparent solar time in the case of the sun or the sidereal time if a star is observed.

We have here neglected the effect of refraction, which, amounting as it does to about 36′ on the horizon, will cause stars to be just visible when they are really 36′ below the horizon.

To find the azimuth of the body, we have

$$\cos S P = \cos S A \cos P A,$$

or

$$\cos S A = \frac{\sin \text{dec.}}{\cos \text{lat.}}.$$

Examples.

1. At a place in lat. 35° S., the bearing of a wall is 110°. Find the apparent time at the equinox when it casts no shadow.

Ans. 3 hrs. 50 min. 24·5 sec. p.m.

2. Find the true bearing and apparent time of sunrise in lat. 32° S. when the sun's declination is 20° S. (Take the sun's centre and neglect refraction and parallax.)

Ans. Bearing, 113° 47′ 05″.
Time, 5 hrs. 07 min. 25 sec.

3. Rigel was observed East of the meridian on the horizontal wire of a theodolite at 7 hrs. 05 min. 20 sec. p.m. by a watch which is supposed to keep West Australian standard time (120th meridian). It was also observed at the same altitude West to cross the horizontal wire at 1 hr. 25 min. 30 sec. a.m. Neglecting the rate of the watch, find its error.

Date of first observation, ...	January 5th, 1908.
Longitude of locality, ...	115° 50′ 26″ E.
Sidereal time at G.M.N., January 5th, .	18 hrs. 54 min. 45·83 sec.
Sidereal time at G.M.N., January 6th, .	18 hrs. 58 min. 42·39 sec.
R.A. of Rigel, ...	5 hrs. 10 min. 07·29 sec.

Ans. 16 min. 9·8 sec. slow.

4. On July 16th, 1910, in latitude 33° 15′ 13″ S. and longitude 10 hrs. 04 min. 50 sec. E., the observed altitude of the sun's centre was 31° 54′ 45″ bearing 10° 35′ 15″ magnetic, the referring mark bearing 86° 54′ 15″ magnetic, time by watch being 10 hrs. 48 min.

The sun's declination at noon on July 15th at Greenwich was 21° 38′ 18″ N., and the mean hourly difference 23·05″ decreasing.

The equation of time to be added to apparent time is 5 min. 46·18 sec., and the hourly increase 0·25 sec.

Find the true bearing of the referring mark, the magnetic variation, and the error of the watch.

Ans. Bearing, 98° 34′ 46″.
Variation, 11° 40′ 31″ E.
Watch error, 3′ 04·3″ fast.

5. At a place 40° 51′ 20″ S., 140° 20′ 30″ E., at 9 hrs. 10 min. 20 sec. a.m. by a watch on 2nd September, 1910, the sun's preceding limb was found by compass bearing to be 58° 14′ 20″, and the observed altitude of the upper limb 27° 11′ 15″.

Declination at G.M.N., September 1st, 8° 31′ 00·7″ N.; hourly variation, 54·24″.

Declination at G.M.N., September 2nd, 8° 09′ 14·4″ N.; hourly variation, 54·58″.

Sun's semi-diameter, G.M.N., September 1st, 15′ 52·61″.
,, ,, September 2nd, 15′ 52·84″.

Equation of time (to be added to apparent time), G.A.N., September 1st, 9·04 sec.

Equation of time (to be subtracted from apparent time), G.A.N., September 2nd, 9·66 sec.

What was the declination of the compass and the correct mean time of observation?

Ans. Declination, 9° 21′ 15″ West.
Mean time, 9 hrs. 07 min. 57 sec.

6. At a place in latitude 32° S. a vertical rod 6 feet high casts a shadow 15 feet long in a direction bearing 75° 12′. What is the apparent time and the approximate time of year?

Ans. 5 hrs. 5½ min. p.m.
December.

7. If the time be found by a single altitude, show that a small error in the latitude will have no effect on the time when the body is in the prime vertical.

8. At 5 p.m. by watch on September 8th at a place in latitude 31° 57′ 08·4″ S., longitude 7 hrs. 43 min. E., the observed altitude of the sun's centre (corrected for instrumental errors) was 29° 58′ 25·2″. Sun's declination at G.A.N., September 8th = 5° 45′ 55·9″ N., variation in one hour 56·40″.

Equation of time to be subtracted from apparent time = 2 min. 18 sec.

Find the sun's true bearing and the error of the watch on West Australian standard time (120th meridian).

Ans. Bearing, 299° 49′ 06·32″.

9. On January 3rd, 1914, at a place latitude 30° 15′ S., longitude 148° E., the following sun observation was taken :—

Observed Altitude.	Alidade Bubble.		Approximate Local Mean Time by Watch.	Angle from R.M.
	E.	O.		
$\vert\overline{\odot}$ 38° 07′ 15″	3	7	8 hrs. 6 min. a.m.	112° 14′ 40″
$\underline{\odot}\vert$ 39° 18′ 37″	2	8	8 hrs. 10 min. a.m.	114° 51′ 20″

Magnetic bearing of R.M , 200° 10′ 20″.

Bubble divisions on Alidade = 20″.

Required : Magnetic Variation and Error of Watch.

Data from Nautical Almanac :—

Sun's Declination.	Hourly Variation.
Jan. 3rd, G.M.N., 22° 53′ 02·4″ S., . . .	14·08″
Jan. 4th, G.M.N., 22° 47′ 11·0″ S., . . .	15·21″

Equation of time (to be added to apparent time).

Jan. 3rd, G.M.N., 4 min. 23·81 sec., . .	1·162″
Jan. 4th, G.M.N., 4 min. 51·51 sec., . .	1·145″

Ans. Magnetic variation = 9° 44′ 11″ E.
Error of watch = 6 min. 42 sec. slow.

CHAPTER XI.

DETERMINATION OF LONGITUDE.

The difference of longitude between any two places on the earth's surface, as we have already seen, is measured by the difference between either their local sidereal times or their local mean times at the same instant. The problem, then, of the determination of the difference in longitude between A and B amounts to that of the determination of the difference in the local times at A and B. By the methods we have considered in the last chapter we may by astronomical observation determine the local time at A at some instant, and a means must be found of determining what is the local time at B at the same instant, if we are to ascertain the difference of longitude.

The problem presented is usually that of the determination of the *difference* of longitude between two places rather than the fixing of the absolute longitude of a place as measured from the now universal standard meridian, that of Greenwich. Usually we seek to find the difference in longitude between a point on a survey and some fixed observatory in the country or some other point on the survey, the longitude of which has been previously determined.

In all cases the local time at some instant must be determined at the place whose longitude is required by one of the astronomical methods of the last chapter. The corresponding local time at the reference station is then in modern practice usually found by one of three ways :—

(*a*) By portable chronometers.
(*b*) By electric telegraph or wireless telegraphy.
(*c*) By flash-light signals.

(*a*) **By Portable Chronometers.**—Since the time when chronometers that will retain a fairly uniform rate have been generally available, this has been the general method for the determination of longitude at sea. Every ship carries a chronometer, which keeps either Greenwich time or the local time at some known port, and from an astronomical observation the Captain is thus able to ascertain the difference between his local time and that of the chronometer. The method is very simple and convenient, but wireless telegraphy, which is capable of much greater precision, may perhaps largely supersede it in the near future. To obtain accurate results it is essential that the chronometer should keep a constant rate, and the conditions on board a ship are much more favourable for this than is usually the case when chronometers are carried about from place to place on land. So that for land work the box chronometers used at sea are commonly replaced by chronometer watches which are more easily carried and are found to be more satisfactory.

Suppose now that it is required to determine the difference in longitude between A and B. The watch or chronometer must first be regulated at station A. Its error on the local time at that place must be determined and its "rate"—*i.e.*, the amount that it gains or loses in 24 hours—must be found. On the assumption that the rate remains constant this will enable the local time at A to be found from a reading of the chronometer at any time afterwards. If then the chronometer be transported to B and an astronomical observation be made there for the determination of local time, it will be possible to find from the chronometer the local time at A at the same instant.

EXAMPLE.—At A, September 8th, 1914, the chronometer at 8 p.m. was found to be 2 min. 6·5 sec. fast, and it was *gaining* at the rate of 2·58 sec. in 24 chronometer hours.

At B, September 9th, 1914, from an astronomical observation which gave the local time as 9 hrs. 12 min. 35 sec. p.m., the reading of the chronometer was 9 hrs. 12 min. 30·6 sec.

What is the difference of longitude?

The interval of time, as measured on the chronometer, between the two readings is 25 hrs. 10 min. 24·1 sec. = 1·049 days.

Therefore, in this interval the chronometer has gained 1·049 × 2·58 sec. = 2·7 sec.

Thus, at B the chronometer was fast by 2 min. 9·2 sec., and the local time at A was 9 hrs. 10 min. 21·4 sec., corresponding to the local time of 9 hrs. 12 min. 35 sec. at B.

Thus, the time at B is in advance of that at A by 2 min. 13·6 sec., or B is to the *East* of A by 0° 33′ 24″.

The accuracy of the method is affected by the fact that the rates of chronometers are not perfectly constant, and particularly by the fact that the rate whilst being carried is not the same as when at rest. The best way to minimise the error is to use several chronometers, from each of which a longitude determination is obtained, and the average of the results is taken. If possible, after the observations have been made at B, the chronometers should be carried back again to A and another comparison made with the local time there.

This method is now never used by surveyors except where telegraphic communication is not available.

(*b*) **By Electric Telegraph or Wireless Telegraphy.**—If two places are connected by electric telegraph the difference of longitude may be obtained with great accuracy.

Suppose that A and B are two stations so connected, A being to the east of B, so that the local time at A is in advance of that at B.

Then if an operator at A taps a telegraphic key that produces a corresponding tap in a telegraphic key at B, the two taps will be very nearly simultaneous, but not quite. A certain slight interval of time, a fraction of a second, will be required to transit the electric current from A to B and to produce the motion of the recording instruments. But whether the signal be transmitted from A to B or in the reverse direction from B to A, the time taken in transmission will be the same.

If now the operators at A and B note the exact instant of each tap on chronometers keeping local time, either mean solar or sidereal, the difference in the times would at once give the difference in longitude if the taps were absolutely simultaneous.

But, actually, when the message is sent from A to B, owing to the time taken in transmission, the tap at B will be a little later than it should, and the result obtained for the difference in longitude will be correspondingly too small.

And similarly when the message is sent from B to A, the tap at A will be made later than should be the case if the transmission were instantaneous, and A being to the east of B, the difference of time will now appear too great.

Thus by averaging the results of sending messages in opposite directions a correct value is obtained for the difference in longitude, and the error due to the time of transmission is completely eliminated.

With signals sent by wireless telegraphy the velocity of the electric wave is so great that practically there is no measurable difference in the results obtained, whether the signals are sent from A to B or from B to A.

For the most refined determinations the signals as received are automatically recorded on a chronograph, but very good work can be done by noting the times of signals with a chronometer if proper methods are adopted.

Recording and Receiving Signals.—A set of signals usually consists of a series of taps made at intervals of 10 seconds by a *sidereal* chronometer, the set extending over from 3 to 5 minutes. Each set is ushered in by a warning rattle of the key. The exact time of each tap is recorded at the receiving station by an observer who is counting out the ticks, which represent half seconds, on a chronometer keeping *mean time*. If the tap occurs between 1·5 and 2·0 seconds, the observer judges whether the time is 1·6, 1·7, 1·8, or 1·9.

It is a very important aid to accuracy that the 10 second signals should be sent by means of a sidereal chronometer and recorded by a mean time chronometer. If the chronometer at the sending and receiving ends kept the same kind of time, the taps would always occur at the same decimal of a second, and the recorder, after the first two or three taps, would probably become prejudiced in favour of some particular value of the decimal which he would retain throughout the set. But if one chronometer keeps sidereal and the other mean time, the tick of the sidereal chronometer will coincide with that of the mean time chronometer every three minutes, and in the interval between the coincidences the decimals of a second recorded at the receiving station will range from ·1 to ·9, so that the judgment of the recorder is not likely to be prejudiced in the same way as it would be if both instruments kept the same kind of time.

Comparison of Chronometers.—If two chronometers keeping the same kind of time, both beating half seconds, are to be compared, it will generally happen that the ticks of the one do not exactly coincide with the ticks of the other, but differ by some fraction of a half second that must be estimated by ear. It is difficult and requires considerable practice to make this estimate nearer than the fifth of a second. But it is possible to compare a sidereal and a mean time chronometer with much greater accuracy, because at intervals of about three minutes the ticks of the two exactly coincide, and, if the comparison be made at the moment of coincidence, there is no difference of a fraction of a beat for the ear to estimate. Thus the difference in the readings of the two chronometers at this particular instant may be obtained exactly. The only error will be that which arises from judging the beats to be in coincidence when they are really separated by a small fraction. But it is found that a difference between the beats as small as 0·02 second is sufficient to enable the practised ear to detect the departure from exact synchronism and consequently the comparison may be made with an error not exceeding this quantity.

The error of the sidereal chronometer is first obtained by astronomical observation, in the manner described in the previous chapter. Then to determine the error of the mean time chronometer a comparison is made at one of the moments when the beats coincide. Listening to the beats of the two chronometers the observer judges when a coincidence is about to occur. He then begins to count the beats of one chronometer while he watches the face of the other. When he no longer perceives any difference in the beats, he notes the corresponding half seconds of the two instruments. The observed instant on the sidereal chronometer is then reduced to mean time, after allowing for the error of the chronometer, and the difference between the result and the recorded instant on the mean time chronometer gives its error.

Personal Equation. — It is found that different men, when performing such operations as sending or recording signals, will differ appreciably in their work. One man, when pressing down a telegraphic key at the instant the chronometer ticks, will consistently do so a little too late. Another will invariably press the key a small fraction of a second too soon. Similarly when recording the time signals one observer will consistently make a larger error than the other. It is found that the more practised and experienced the observers are, the more regular and consistent are the errors made in this way, and that this personal error or "personal equation," as it is commonly called, remains fairly constant for long periods of time. Consequently its effects may be largely eliminated, in the average of a considerable number of observations, if the personal equations of the observers be determined both before and after the observations are made.

In this case the relative personal equation is required between two observers. It may be most simply obtained by the observers setting up their instruments near to one another at the same station. They then send sets of signals to one another, just as they would do in ordinary field work, in order to determine their difference of longitude. This should be done under conditions as nearly as possible the same as those obtaining at the actual work in the field. The result obtained, which should of course be zero, is the relative personal equation that must be applied in the reduction of the field observations. It is advisable to observe the personal equation in this way for two or three evenings shortly preceding and following the field trip.

When a large number of observations is being made probably the best way of eliminating the error due to personal equation is to exchange the observers at the ends of the telegraph line when half the total number of signals have been transmitted. When A sends and B receives, the time recorded at the receiving station should exactly coincide with the time of sending. Usually it does not, owing to the existence of this personal equation, and the time actually recorded by B may be either before or after the chronometer tick that A is transmitting.

If the time recorded is always *after* the chronometer tick, the error will be fairly consistent so long as A is sending and B receiving. If B is at a station to the east of A, the effect of this error will be to make the difference of longitude greater than it really is, but if B is at a station to the west of A the same error will make the difference of longitude appear less than it should be. Thus if the observers change places when half the observations are over, personal equation is eliminated in the mean of the whole set and there is no necessity to make a special determination of it.

Programme of Operations.—Observations are made on several evenings. Professor W. E. Cooke, who was responsible for the introduction of the almucantar method of time observation in Western Australia, thus summarises the operations for any one evening :—

Observations.

(*a*) Compare sidereal and mean time chronometers.

(*b*) Take first half of almucantar observations, using sidereal chronometer.

(*c*) Take chronometers to telegraph station and exchange signals sending from sidereal and receiving by mean time.

(*d*) Complete almucantar observations.

(*e*) Compare the two chronometers.

Computations.

(*f*) From the almucantar observations determine the error of the sidereal chronometer at some definite sidereal hour, also its rate.

(*g*) Apply the rate so as to obtain the error at time (*a*); reduce sidereal time (*a*) to mean, and hence determine error of mean time chronometer at time (*a*).

(*h*) Do the same for time (*e*).

(*i*) From (*g*) and (*h*) determine the errors of each chronometer at time (*c*).

(*j*) Apply these errors to the average of the signals, also apply the correction for personal equation. Subtract the results from the similar results at the other station, and thus the difference of longitude will be obtained.

When a determination of difference of longitude is made telegraphically between fixed observatories, the precision of the method is increased by sending the signals from a clock, the pendulum of which automatically completes an electric circuit when at the bottom of its stroke. The record at the other station is then taken on a chronograph, from which the instant can be read off to the hundredth part of a second. Such equipment is, however, not usually available for field work.

(*c*) **By Flash-Light Signals.**—When two stations are visible one from the other, flash light signals may be sent from one at ten second intervals as determined by the tick of a sidereal chronometer and recorded at the other by means of a chronometer keeping mean time, just as with electric telegraph signals. Or the signals may be sent from an intermediate station that is visible from both. The observers at each station must of course have obtained their local time by proper observation, and the difference between their local times at the instant of the signal gives at once the difference of longitude. The signal may be made by the flash of a heliotrope by day or the eclipse of a bright light at night.

The following examples gives the results of observations made in this way in Western Australia to determine the difference of longitude between the Perth Observatory and Mount Maxwell, about 17 miles away to the east. The signals were made by means of an acetylene lamp placed in a box, the light shining through a hole over which a photographic snap-shutter was fixed. The shutter was released at the proper second and the time of the flash noted as it was seen through a theodolite at the other station. The example is taken from the Western Australian Handbook for Surveyors :—

DIFFERENCE OF LONGITUDE.

1909.	Mount Maxwell to Observatory.	Observatory to Mount Maxwell.	Mean Result.
Nov. 6th, . .	1′ 7·96″	1′ 8·64″	1′ 8·30″
Nov. 7th, . .	1′ 7·87″	1′ 8·56″	1′ 8·21″
Nov. 8th, . .	1′ 7·82″	1′ 8·54″	1′ 8·18″
Nov. 9th, . .	1′ 7·81″	1′ 8·61″	1′ 8·21″
Nov. 13th, . .	1′ 7·93″	1′ 8·53″	1′ 8·23″
	Mean, ...		1′ 8·25″
	Personal equation, . .		+0′ 0·06″
	Difference of time, . .		1′ 8·31″

Longitude by Lunar Observations.—The methods for the determination of longitude that have just been described are those nowadays most usually adopted, but before the invention of the electric telegraph and the perfection of chronometers the only methods available over long distances depended upon observations of the moon. The moon changes its position among the fixed stars much more rapidly than any other celestial body, its relative movement amounting to over 13° in 24 hours, or roughly it moves over a distance equal to its own diameter in one hour. Consequently it is possible to use it as a clock, and, by measuring its position with regard to surrounding stars, we may determine at any instant, with the aid of the tables of the moon's motion given in the Nautical Almanac, the corresponding time at Greenwich. It was chiefly in order that the moon's motion might be systematically observed for the purpose of providing navigators with accurate tables, which could be used for the determination of longitude, that the Greenwich observatory was originally founded. Lunar observations, however, generally entail rather laborious computation, and the results, with the exception of those obtained by the method of lunar occultations, are not comparable in accuracy with the determinations made by the simpler methods previously given. Consequently such methods are now rarely used on land, and we shall merely describe the general principles involved.

There are three principal methods of making observations upon the moon for longitude. They are :—

(*a*) By Lunar Distances.
(*b*) By Lunar Culminations.
(*c*) By Lunar Occultations.

(*a*) *By Lunar Distances.*—The angular distance between the bright limb of the moon and some bright star in its vicinity is measured by means of the sextant, and at the same instant the altitudes of both moon and star are observed. This is best done by three observers, one for each measurement, but if there is only one observer, he takes first the altitudes, then the lunar distance, and then the altitudes once more, noting the time of each observation. From these he readily deduces the proper altitudes at the moment when the lunar distance was measured.

By adding or subtracting to the observed distance the apparent semi-diameter of the moon, according as the bright limb of the moon is toward or from the star, the *apparent distance* between the star and the moon's centre is found. The moon's semi-diameter is given on page 3 of each month in the Nautical Almanac, for noon and midnight of each day. From this apparent distance, allowing for refraction and parallax, and knowing the approximate latitude of the place, the observations enable the distance to be computed as it would be observed from the centre of the earth, or the *true distance* as it is commonly termed. But if we know the true distance the corresponding time at Greenwich may be found from the information given in the Nautical Almanac. And the local time of the observation is readily found from the observed altitude of either moon or star. The longitude is found, of course, as the difference between the local time and the corresponding Greenwich time.

In fig. 49 let S and M denote the apparent positions of the star and the moon's centre respectively, Z being the Zenith. Parallax and Refraction will affect them in the vertical planes Z S and Z M. Now refraction causes a body to appear at a higher altitude than it really has, whilst a body when viewed from the earth's centre will have a greater altitude than when seen from the earth's surface. Thus to allow for refraction we have to decrease the observed altitude, and to allow for parallax we must increase it. Now in the case of the moon parallax is greater than refraction, the contrary being true for a star or planet. Thus the "true" position of S, as observed from the earth's centre, is at S_1, below S, and the true position of M is at M_1, above M.

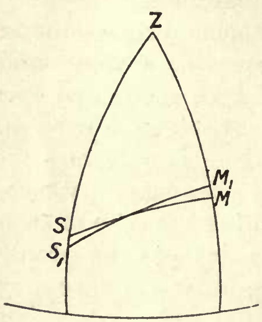


Fig. 49.

In the triangle Z S M, the three sides have been directly determined by observation, and, therefore, the angle Z may be computed by the ordinary rules of spherical trigonometry. Then in the triangle Z S_1 M_1, Z S_1, and Z M_1, are known, and also the included angle Z, consequently the *true distance* M_1 S_1 may be computed.

The Nautical Almanac used to give a table of true lunar distances, for every third hour of Greenwich mean time, from selected suitable bright stars. But these tables have lately been discontinued as it was decided that they were no longer of sufficient use to warrant their retention.

The method is not capable of any degree of precision, about 5 seconds of time representing the accuracy attainable, and, now that the tables of lunar distances are no longer published, involves a lot of computation. The measurements cannot be made by a theodolite, the sextant being essential, and the method can only be classed as a rough one under the best circumstances.

(*b*) *By Lunar Culminations.*—As the moon moves right round the earth in a lunar month of about 28 days, its right ascension must change by 360° in that period, or at an average of about 13° in 24 hours. Thus in one hour its right ascension will alter on the average by something over 30 minutes of arc or two minutes of time. Now the right ascension of the moon may be most easily measured by observing the difference in time between its transit across the meridian and that of some known star. If the local time at the place of observation is also known, this determines the right ascension of the moon at a given instant of local time. But the Nautical Almanac gives the right ascension of the moon for every hour of Greenwich time throughout the year, and, by interpolation between the values in the tables, the Greenwich time corresponding to the measured right ascension may be found. Then the difference between the local time of observation and the corresponding Greenwich time as thus determined gives the longitude required. The computations are thus simple, and the method is the easiest of all the lunar methods for finding longitude.

The observations are facilitated by the tables of moon-culminating stars given in the Nautical Almanac on p. 412 and succeeding pages. In these tables for each day in the year there are tabulated one or two stars, known as moon-culminating stars, that do not differ much from the moon in either right ascension or declination, and are consequently suitable for meridian transit observations in comparison with the moon. For if the declination of the observed star does not differ much from that of the moon, any error in the setting out of the meridian will affect the times of both transits to the same extent, and in the difference between the two times of transit, which is what is sought, the error will be eliminated.

The times of meridian transit are unaffected by parallax and refraction which introduce complications in other lunar methods. A disadvantage is that for a considerable part of the month transits occur at very inconvenient times.

The method in any case is not capable of great accuracy. An error of one second in the measurement of the time of transit of the moon's limb will cause an error of about 30 seconds of time in the longitude. Thus a good observation will only determine the longitude within about 10 seconds of time, and only by the average of a number of careful observations will it be possible to determine the longitude by this method within 5 seconds of time, corresponding to $1\frac{1}{4}$ minutes of arc, or to a distance of over one mile near the equator.

EXAMPLE.—*At a place in approximate longitude* 9 *hrs.* 06 *min. E. the times of transit across the meridian of the moon's bright limb and of the star σ Aquarii were recorded by means of a chronometer keeping local mean time on the evening of September* 30*th*, 1914.

*Observed time of transit of Moon I.**, .	9 *hrs.* 14 *min.* 22·8 *sec.*
„ „ *σ Aquarii*, .	9 *hrs.* 52 *min.* 30·2 *sec.*

Determine the longitude of the place.

Difference in times of transit, . .	38 min. 07·4 sec.
Equivalent interval of sidereal time, .	38 min. 13·66 sec.
R.A. of σ Aquarii, ...	22 hrs. 26 min. 09·87 sec.
R.A. of Moon I., ...	21 hrs. 47 min. 56·21 sec.

* The Roman numerals I. and II. are used in the Nautical Almanac to indicate the moon's preceding and following limbs respectively.

Allowing for the approximate longitude, the transit takes place at about 8 minutes after Greenwich noon on September 30th.

From the Nautical Almanac we obtain

	Time of Meridian Passage at Greenwich.	Sidereal Time of Semi-diameter Passing Meridian.
Sept. 30th, . .	9 hrs. 32·1 min. (upper)	63·92 seconds
Sept. 29th, . .	21 hrs. 10·1 min. (lower)	65·02 ,,

Thus, the sidereal time for the semi-diameter to pass the meridian is given by

$$63{\cdot}92 + \frac{9 \text{ hrs. } 24 \text{ min.}}{33 \text{ hrs. } 32 \text{ min.} - 21 \text{ hrs. } 10 \text{ min.}} \times 1{\cdot}1 = 64{\cdot}77 \text{ sec.}$$

∴ R.A. of moon's centre at instant of observation

= 21 hrs. 49 min. — 00·98 sec.

Again, from the Nautical Almanac,

R.A. of moon at Greenwich, 0 hr. = 21 hrs. 48 min. 44·20 sec.

,, ,, 1 hr. = 21 hrs. 50 min. 41·41 sec.

Therefore, by interpolation, the Greenwich mean time corresponding to the R.A. of 21 hrs. 49 min. 00·98 sec. is

0 hr. 08 min. 35·4 sec.

But the observed local time of the observation is

9 hrs. 14 min. 22·8 sec.

Therefore, the longitude is 9 hrs. 05 min. 47·4 sec. East.

(*c*) *By Lunar Occultations.*—In the course of its monthly revolution round the earth the moon covers or "occults" in turn a number of the fixed stars. As the moon apparently moves from West to East among the stars, the stars in its track first disappear under the Eastern limb and afterwards reappear on the other side. The covering of a star in this way by the moon is known as an "occultation," the disappearance of the star behind the Eastern limb of the moon being known as the "immersion," and its reappearance as the "emersion." The method by lunar occultations consists in observing the local time of immersion or emersion, or both, at the occultation of a known star. At such moments the apparent right ascension of the star is the same as that of the Eastern or Western limb of the moon, and, after making proper allowance for refraction, parallax, and semi-diameter, the true right ascension of the moon may be determined at the instant, and hence, from the tables in the Nautical Almanac, the corresponding Greenwich time may be found.

The method is capable of much greater accuracy than any other method by lunar observations. The two methods previously described, even under the most favourable conditions, can give but roughly approximate results. But from several observations of lunar occultations a longitude may be determined within less than one second of time. Unfortunately, however, the prediction of the circumstances of an occultation and the complete computation of the observations involve principles that are rather complex for an elementary work. Partly on this account, and partly because suitable observations can only be made at any one place some three or four times in a month as a rule, the method is not one used to any extent by surveyors, and no further elaboration of the method will in consequence be attempted here.

Relative Accuracy of Different Methods.—Major Close, in his *Text Book of Topographical Surveying*, gives the following table showing the terminal error in longitude which might be expected after a march of 300 miles in a hilly tropical country.

Method.	Probable Error in Longitude.
Triangulation,	100 yards to $\frac{1}{4}$ mile.
Telegraph,	$\frac{1}{8}$ to $\frac{1}{4}$ mile.
Chronometers,	1 mile.
Occultation,	$\frac{1}{4}$ mile.
Moon culminations,	1 mile.
Lunar distance,	10 miles.

The probable errors are here stated as distances measured parallel to the equator, but, as the actual measurements of longitude are made in time, and as the distance measured along the earth's surface corresponding to a given difference of time gets less and less as we proceed further from the equator, it follows that the probable errors in distance would be considerably less than those chronicled at places remote from the equator.

Where a triangulation can be carried on to directly connect the two places whose difference of longitude is required, the determination may be made with the greatest precision possible. The telegraphic method comes next in order of accuracy, and is nowadays the method most commonly used. In order to get anything like the same accuracy by the method of lunar occultations, the observations would have to extend over several months, and the tabulated values for the right ascension of the moon given in the Nautical Almanac would have to be corrected from observations made at some fixed observatory.

CHAPTER XII.

THE CONVERGENCE OF MERIDIANS.

THE line of sight of the telescope of a theodolite in accurate adjustment, as the telescope is turned about its horizontal axis, traces out a vertical plane. This, if we regard the earth as spherical, we may consider to be a plane passing through the centre of the earth. Therefore, the straight line that is set out by a theodolite is in reality always the arc of a great circle on the earth's surface. Now, unless it happens to coincide with the equator or with a meridian of longitude, any great circle will cut different meridians at different angles. In other words, its bearing will vary from point to point. Thus as we proceed along a straight line set out by a theodolite on the earth's surface, the bearing of the line will not remain constant but will gradually alter. A line the bearing of which was everywhere the same would not be a straight line. A parallel of latitude for instance is such a line, but if the telescope of a theodolite is set out truly East and West at any place its direction would not mark out the parallel of latitude, which is a small circle, but a great circle that would ultimately intersect the equator.

This alteration in the bearing of a straight line is an important matter in surveys of any magnitude, as in latitudes in the neighbourhood of 60° it amounts to considerably over a minute of arc in a line one mile long, and in higher latitudes the alteration is still greater.

In fig. 50, let N and S denote the North and South terrestrial poles, E L M Q is the equator, and A and B any two points between which the great circle arc A B has been set out.

Let N A M S and N B L S be the meridians through A and B. Then the bearing of the line B A at B is the angle N B A, and the bearing of the same line at A is $180° - \text{N A B}$.

The difference between the bearings of the line A B at the points A and B is known as the *convergence* of the meridians between A and B.

If A B is plotted as a straight line on a plane, then the

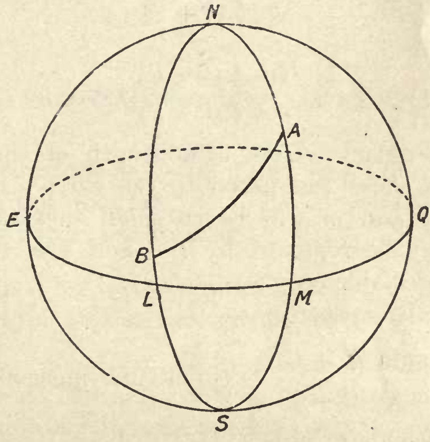


Fig. 50.

meridians through A and B will not be drawn as parallel lines, but as lines making an angle with one another equal to the convergence.

Denote the convergence by c.

Then $c = 180° - \text{N A B} - \text{N B A}$.

Let $l =$ latitude of A and $l' =$ latitude of B.

$\text{N A} = 90° - l$, $\text{N B} = 90° - l'$.

Denote the difference of longitude between A and B by m, so that $m =$ angle B N A.

Then in the spherical triangle N B A, having given two sides and the included angle,

$$\tan \tfrac{1}{2}(\text{N B A} + \text{N A B}) = \frac{\cos \frac{1}{2}(\text{N B} - \text{N A}) \cot \frac{1}{2} m}{\cos \frac{1}{2}(\text{N B} + \text{N A})}.$$

$$\therefore \quad \cot \tfrac{1}{2}(180^\circ - \text{N B A} - \text{N A B}) = \frac{\cos \frac{1}{2}(l - l') \cot \frac{1}{2} m}{\cos \frac{1}{2}(180^\circ - l - l')},$$

$$\therefore \quad \cot \tfrac{1}{2} c = \frac{\cos \frac{1}{2}(l - l')}{\sin \frac{1}{2}(l + l')} \cot \tfrac{1}{2} m,$$

or, inverting

$$\tan \tfrac{1}{2} c = \frac{\sin \frac{1}{2}(l + l')}{\cos \frac{1}{2}(l - l')} \tan \tfrac{1}{2} m.$$

In any ordinary survey, the length of the line A B will be very small compared to the earth's radius, and the angles c and m will be so small that $\tan \frac{1}{2} c$ and $\tan \frac{1}{2} m$ may be replaced by $\frac{1}{2} c$ and $\frac{1}{2} m$ respectively without appreciable error.

$\therefore$ c (in circular measure)

$$= \frac{\sin \frac{1}{2}(l + l')}{\cos \frac{1}{2}(l - l')} m \text{ (in circular measure)},$$

and c (in seconds of arc)

$$= \frac{\sin \frac{1}{2}(l + l')}{\cos \frac{1}{2}(l - l')} m \text{ (in seconds of arc)}.$$

Again, unless the line A B is a very long one, $\cos \frac{1}{2}(l - l')$ differs from unity by but a very small quantity, so that for ordinary purposes

Convergence in seconds = sin mid. lat. × diff. of long. in seconds.

Another convenient form of the result expresses the convergence in terms of the "departure" between A and B; that is to say, their distance apart measured in an East and West direction.

The parallel of middle latitude is a circle of radius $r \cos \frac{1}{2}(l + l')$, where r is the radius of the earth in miles, and, therefore, if d denotes the departure in miles,

$$\frac{d}{r \cos \frac{1}{2}(l + l')} = \text{the circular measure of } m.$$

$\therefore$ convergence in seconds

$$= \sin \tfrac{1}{2}(l + l') \frac{d}{r \cos \frac{1}{2}(l + l') \sin 1''}$$

$$= \frac{d \tan \frac{1}{2}(l + l')}{r \sin 1''}.$$

Taking r as 3,958 miles we obtain, therefore, the following rule :—

To the constant log, ...	1·7169
Add log tan mid. lat., . . .	
Add log departure in miles, . . .	
The sum is log of the approximate number of seconds in the convergence, .	

Thus for a departure of 1 mile in latitude 20°, the convergence is 19″ only, but in latitude 40° it is 44″, and in latitude 60° it is as much as 90″.

It thus appears that the convergence increases very rapidly in high latitudes, and that in latitude 60° the bearing of a straight line one mile long and running approximately E. and W. will at one extremity be different by 1·5 minutes from what it is at the other.

The amount of convergence is such that when a straight line is run several miles in length the bearing of the line as determined by astronomical observation will differ appreciably at each end. The nearer the place is to the equator, the longer the line will have to be before the difference is sufficient to directly observe. In latitude

40° it is readily observable at the end of an East and West line two miles long, in latitude 60° the line need be only one mile long for the difference to be just as readily detected. There is no such effect in lines running directly N. and S., as such lines form a part of a meridian of longitude, and the convergence is greatest at the extremities of lines of given length, when the direction is E. and W.

The investigation we have given for convergence is of course an approximate one only, and the formulæ obtained are not exact, because the earth is not in reality a true sphere as has been assumed. The results obtained, however, are quite sufficiently accurate for all but the most refined geodetic work.

Miscellaneous Examples.

1. At what height would a signal need to be erected at station B to be visible from the instrument at A, so that the line of sight would be 10 feet clear of the summit of an intervening hill at C ?

Height of instrument above sea level at A, 488 feet. Station B, 20 miles distant from A, 5·2 feet. The summit of the intervening hill, 12 miles from A, 442 feet.

Ans. 32·7 feet.

2. A man on a height near Pietermaritzburg, 42 miles from Durban, owing to the clearness of the air can see a ship 6 miles out at sea. Looking in the other direction he can see the heights of Drakenburg, which he knows are 110 miles from him. Find the height of the Drakenburg above the sea, taking the radius of the earth as 3,960 miles. (*Educational Times.*)

Ans. Half a mile nearly.

3. From a point in latitude 30° South, longitude 120° East, a line at right angles to the initial meridian is run Easterly for a distance of 18 miles. Find the true bearing of the line at its Easterly end, its longitude, and the bearing and distance to a point in that longitude in the same latitude as the starting point. Assume the radius of the earth to be 3,960 miles.

Ans. (*a*) 269° 50′ 59″.
(*b*) Longitude, 120° 18′ 02″.
(*c*) Due South, ·024 mile.

4. On the evening of the 12th April, 1911, the altitude at meridian transit of the star *a* Hydræ, North of the Zenith was observed from two hills,

A and B, a considerable distance apart. Altitudes of α Virginis, in the eastern sky, were observed simultaneously from both hills by aid of prearranged signals. Several sets were taken, which, reduced to a mean and cleared of corrections for refraction and level errors, gave the following results :—

At station A the meridian altitude of α Hydræ was 63° 22′ 40″ and the altitude of α Virginis was 12° 14′ 18″.

At station B the meridian altitude of α Hydræ was 63° 44′ 40″ and the altitude of α Virginis was 12° 44′ 18″.

The declination of α Hydræ was 8° 16′ 26″ S., and the declination of α Virginis was 10° 42′ 0″ S., taken from the Nautical Almanac.

Find the distance between the two hills A and B in miles and decimals, and the true bearing of each station, treating the earth as a sphere having a radius of 3,968 miles.

Ans. Distance = 44·64 miles.
Bearing of B from A,
55° 31′ 04″.
Bearing of A from B,
335° 09′ 02″.

5. What is, approximately, the spherical excess in a triangle on the earth's surface, two sides of the triangle being 163,421 feet and 154,599 feet respectively, and the observed included angle being 60° 05′ 12·32″? What factors do you require for an exact evaluation?

6. In latitude 45° N. an observer sees a certain star rise in the N.E. If the observer travels to another place with a slightly different latitude, show that the change in direction of the same star at rising will be equal to the change in latitude.

7. Show that all the stars observable from any one place have the same rate of change in azimuth at rising.

8. Prove that the rate of change in altitude of a star is always greatest, when the star is in the prime vertical.

INDEX.

A

ABBREVIATIONS, 133.
Alidade level, 95.
—— —— Allowance of error of, 95.
Almucantar method for determination of time, 207-211.
Alternating method for determining local sidereal time, 58.
—— —— for determining local mean time, 60.
Altitude, 14.
—— and azimuth, Determination of, at short interval, 76-79.
—— —— —— having given R.A., declination, latitude and time, 71-74.
—— —— —— of star at elongation, 107.
—— after star is in known position, 71.
—— of celestial pole, 25.
—— of pole star, for latitude determinations, 102.
Altitudes, equal, Method of, correction for sun observations, 205.
—— —— —— of, for azimuth, 98.
—— —— —— of, for time, 202.
—— Meridian, Method of, at both culminations, 155.
—— —— —— of, for latitude, 149-152.
Antarctic circle, 25.
Apparent motion of the stars, 10-13.
—— solar time, 44, 46.
—— —— times at same instant in places of different longitude, 50.
Arctic circle, 25.
Aries, First point of, 15, 43.
—— —— —— of, time of transit, 63-67.
Arrangement of computations, 192, 193.
Astronomical co-ordinates, 13-20.
—— ternio, Synopsis of, 19.
Atmospheric refraction, 83-87.
Autumnal equinox, 43.
Averaging observations, 195-198.
Azimuth, 14.
—— Altitude and hour angle at, 107-111.
—— Method of determination of, 71-79.

B

BESSEL'S formula for refraction, 52.

C

CALCULATION of the time of elongation, 106.
Cancer, Tropic of, 24, 25, 37.
Capricorn, Tropic of, 24, 25, 38.
Celestial equator, 13, 39.
—— sphere, 8.
—— —— To plot position of sun's centre on, 40.
Changing face, Elimination of errors by, 90.
Chronometers, for longitude determination, 220.
—— Comparison of, 223.
Circles, Great, 1.
—— Small, 2.
Circular parts, Napier's rules of, 3-5.
Circum-elongation observations for azimuth, 131-133.
Circum-meridian observations for latitude, 155-160.
—— —— Limits of time for, 160.
—— —— of the sun, 160-163.
Coefficient of refraction, 84.
Collimation, Error of, 88.

Comparative advantages of co-ordinate systems, 17.
Comparison of preceding methods, 62.
Computations, Arrangement of, 118, 192, 193.
Convergence of meridians, 236-241.
Co-ordinates, Astronomical, 13-20.
Corrections, Instrumental, 87.
—— to observations of altitude and azimuth, 80-96.
—— to sun observations, 119.
Culmination, Computation of time of, 162.
—— Lower, of star, 155.
—— Upper, of star, 155.
Culminations, Lunar, for longitude determination, 231.

D

Data necessary for computation, 192.
Daylight, Star observations in, 112.
Declination, 15, 17.
—— circle, 17.
—— Computation of sun's, from Nautical Almanac data, 39, 118, 119.
Degree of longitude, The length of, 23.
Determination of level error of axis by means of the striding level, 93.
—— of true meridian, 97-148.
Distance between places whose latitudes and longitudes are known, 26-30.
Distances, Lunar, for longitude determinations, 229.
—— of stars, 8.

E

Earth, The, figure of, 30-32.
—— —— its shape, 21, 31, 32.
—— —— its orbit round the sun, 21.
—— —— Zones of, 24.
Ecliptic, 43.
—— Obliquity of the, 43.
Effect of an error of collimation, 88.
Effect of an error in direction of prime vertical, 167.
Effect of an error in latitude, 111, 112.
Effect of an error in the longitude of place of observation, 124.
Effect of an error in the measured altitude, 124.
Effect of an error in the sun's declination upon the calculated azimuth, 122-124.
Elimination of instrumental errors by changing face, 90.
Ellipsoid, The earth an, 32.
Elongation, Altitude, azimuth, and hour angle at, 107.
—— Calculation of time at, 106.
—— Observation of star at, 107-111
—— Observation of star near, 109.
Equal altitudes, Method of, for azimuth, 98.
—— —— —— of, for time, 202.
Equation of time, 47.
—— Personal, 224.
Equator, Celestial, 13, 39.
Equinoctial points, 43.
Equinoxes, 38, 39, 43.
Error of transverse axis, Measurement of, 91.
Errors, Instrumental, 90-96.
—— —— Elimination of, by changing face, 90.
Excess, Spherical, 7.
Extra meridian observations, Best time for, 126.
—— —— —— for azimuth, 113-125.
—— —— —— for time, 190-207.
—— —— —— on sun or star, 113-118, 188.

F

Figure of earth, 31.
First point of Aries, 15, 43.
—— —— Time of transit of, 63-67.
First point of Libra, 43.
Flash light signals for longitude determinations, 223.
Formula, Choice of, for extra-meridian observations, 191.
Frigid Zone, 25.

G

GEOCENTRIC latitude, 32-34.
Geographical latitude, 32-34.
Gnomon, 212.
Great circles, 1.

H

HORIZON, Celestial, 10.
Horizontal dial, 212-214.
—— parallax, 82.
Hour angle, 18.
—— —— being known, to determine time, 18, 193.
—— —— of star at elongation, 18.

I

INCLINATION of earth's axis to plane of orbit, 36-38.

L

LATITUDE by altitude of pole star, 169-172.
—— by circum-meridian observations, 155.
—— by horizontal angle between two circumpolar stars at elongation, 175.
—— by meridian altitudes, 149-155.
—— by prime vertical transits, 163.
—— by rate of change of altitude near prime vertical, 163.
—— Effect of an error in, 111-113, 198-200.
—— Geocentric, 32-34.
—— Geographical, 32-34.
—— Methods of determination of, 149-181.
—— Terrestrial, 21.
Length of a degree of longitude, 23.
Libra, First point of, 43.
Local mean time, 49, 55, 70.
—— sidereal time, 50-54, 70.
Longitude by comparison of chronometers, 223.
—— by electric telegraph, 221.
—— by flash-light signals, 227.
Longitude by lunar culminations, 231.
—— by lunar distances, 229.
—— by lunar occultations, 233.
—— by personal equation, 224.
—— by portable chronometers, 220.
—— by recording and receiving signals, 223.
—— Length of a degree of, 23.
—— Methods of determination of, 219-235.
—— Terrestrial, 21.
Lunar observations for longitude, 228-234.

M

MEAN noon, 47.
—— solar time, 46, 47.
—— time, reduction to sidereal, 52.
Meridian, 14.
—— altitudes for determination of latitude, 149-152.
—— altitudes of a star at both lower and upper culminations, 155.
—— Determination of, by circum-elongation observations, 131-133.
—— —— of, by close circumpolar star, 127.
—— —— of, by equal altitudes, 98.
—— —— of, by extra-meridian observations 113, 115, 116, 190.
—— —— of, by star at elongation, 102.
—— —— of true, 97-107.
—— transit, Computation of time of, 162.
—— transits on both sides of Zenith, 187.
Meridians, Convergence of, 236-241.
Methods of determining latitude, 149-181.
—— —— —— longitude, 219-235.
—— —— —— time, 182-218.
—— —— —— true meridian, 97-148.
Motion, Apparent, of stars, 10-13.
—— —— of sun, 35-38.
—— in right ascension and declination, 39.
Moon's motion, 228-234.

N

NADIR, 9.
Napier's rules of circular parts, 3-5.
Nautical almanac, 39, 47.
—— —— data with regard to time, 67-70.
Noon, Apparent, 47.
—— Mean, 47.
North temperate zone, 25.
Notation, 133.

O

OBLATE spheroid, The earth an, 31.
Oblique-angled spherical triangles, Solution of, 5-7.
Oblique dial, 215.
Obliquity of ecliptic, 43.
Observations on both East and West stars, 195-198.
Occultations, Lunar, for longitude determination, 233.

P

PARALLAX, 80-83.
—— Horizontal, 82.
Parallel of latitude, 23.
Personal equation, 224.
Plot position of sun's centre on celestial sphere, 40-42.
Polar distance, 17.
Pole, Celestial, 25.
Portable chronometers, 220.
Prime vertical, 18, 163.
—— —— dial, 214.
—— —— transits, for determination of latitude, 172.

R

RECORDING and receiving telegraphic signals for longitude, 223.
Referring mark for azimuth observations, 97-106.
Refraction, atmospheric, Correction for, 83.
Relative accuracy of methods for finding longitude, 234.
Residual instrumental errors, 87.
Right-angled spherical triangles, Solution of, 3.
Right ascension, 15, 16.
—— —— and declination, Motion in, 39.
—— —— of star, To determine, 15, 74, 75.
Rising of a celestial body, Time of, 215.
Rules of circular parts, 3-5.

S

SETTING of a celestial body, Time of, 215.
Sidereal day, 17.
—— time, 17, 44, 46.
—— —— at local mean noon, 54.
—— —— reduction to mean time, 60-62.
Small circles, 2.
Solar time, Apparent, 44.
Solstitial points, 43.
Solution of spherical triangles, 3-7.
South temperate zone, 25.
Sphere, Celestial, 8.
Spherical excess, 7.
—— triangles, Oblique, 5-7.
—— —— Right-angled, 2-5.
Standard time, 51.
—— —— to change to local mean time, 52.
Star observations in daylight, 112.
Stars, Apparent motion of, 10-13.
—— averaging several observations of the same, 195-198.
—— observations on both East and West, 195-198.
Stile of sun dial, 212.
Striding level, Correction to prime vertical observations, 168.
—— —— Correction to time of meridian transit, 129.
—— —— Use of, 93.
Sun, Apparent motion of, 35-38.
—— dials, 211.
—— —— Horizontal, 212.
—— —— Oblique, 215.
—— —— Vertical, 214.
—— Earth's orbit round, 36.
—— Motion in R.A. and declination of, 39.

Sun, observations, 116-118.
—— Semi-diameter of, 39.
Sun's apparent annual path, 42,43.
—— —— motion among the stars, 35, 36.
—— centre in celestial sphere, To plot position of, 40-42.
—— declination, Computation of, 39, 118, 119.
Synopsis of astronomical terms, 19.

T

TABLES for reduction of circum-meridian observations, 137, 176, 177.
Telegraphic signals for longitude determination, 223.
Temperate Zone, 25.
Terrestrial latitude and longitude, 21-23.
Three systems of time measurement, 46-47.
Time, Apparent solar, 44, 46.
—— determination by almucantar method, 207-211.
—— —— by equal altitude observations, 202.
—— —— by extra meridian observations, 190-207.
—— —— by meridian transits, 62, 182.
—— Equation of, 47.
—— Local mean, 49, 55-70.
—— —— —— reduction to sidereal, 52.
—— —— sidereal, 50, 54-70.
—— —— —— reduction to mean, 52.
—— Mean, 46.
Time measurement, Three systems of, 46, 47.
—— Nautical almanac data with regard to, 67-70.
—— observations upon a close circumpolar star, 127-131.
—— of meridian transit of star, 162.
—— of rising or setting of celestial body, 215.
—— of transit of first point of Aries, 63-67.
—— Standard, of different countries, 51.
Torrid Zone, 25.
Transit, meridian, Time of, 182.
—— of first point of Aries, 63-67.
Transits, Meridian, on both sides of Zenith, 187.
Transverse axis, error due to want of horizontality, 185.
Triangles, spherical, Solution of, 3.
Tropic of Cancer, 24, 25, 37.
—— of Capricorn, 24, 25, 38.
True meridian, Determination of, 97.
Tycho Brahe and the sun's position among the stars, 36.

V

VERNAL equinox, 43

Z

ZENITH, 9.
—— pair observations of stars for latitude, 152-154.
Zone, North temperate, 25.
—— South temperate, 25.
Zones of the earth, 24.

BELL AND BAIN, LIMITED PRINTERS, GLASGOW.